Dynamics of Delegated Search*

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Abstract

In many business contexts such as product design and development, advertising, and scouting for technical solutions or talent, Clients delegate external experts (Providers) to act as their search agents. Providers can control the distribution of search outcomes through the intensity of their search efforts. The Client evaluates the quality of the solutions and decides when to stop the search. The iterative nature of these searches combined with the client's inability to directly observe the provider's actions lead to strategic effort exertion by providers. We explore how the client can use the structural elements of a delegated search to influence the providers' efforts over multiple iterations. In particular, we consider three distinct delegation structures: Committed structure, where the client defines any acceptable solution as a terminal breakthrough; Open-ended structure, where the client reserves the right to continue the search even after receiving an acceptable solution; and Silent Open-ended structure, where the client also does not provide feedback on any intermediate solution.

We analyze the client's and provider's optimal strategic behaviors under these search structures. We show that the provider has an incentive to procrastinate; the level of procrastination can be worse in the Open-ended structure, which is fueled by the difficulty of pleasing the client in the early part of the search. In contrast, the Silent Open-ended search could minimize the provider's procrastination. In addition, our analysis generates insights on the relative efficiency of these structures and how they depend on the project's difficulty and provider's capability. Interestingly, we find that, due to the strategic behavior of the provider, there are many cases where the client benefits by simply committing to a breakthrough structure with full transparency.

Key words:delegation, search structure, procrastination, game theory.

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1 Introduction

In search of creative and innovative solutions to critical problems, firms often seek the expertise of external entities. These solution experts – or providers – are *delegated* by clients to act as their search agents in many business contexts such as product design and development, advertising and marketing, and scouting for technical solutions, or recruiting new talent. The increase in demand for such delegated search services can be witnessed through the rapid growth in revenues of advertising and design agencies (Johnson, 2015). Delegating search to an external provider could have many benefits for clients: it could enable them to access the provider's advanced skills and expertise (Eppinger and Chitkara, 2006), receive a broader set of solution approaches (Erat and Krishnan, 2012), protect the integrity of the search process and reduce bureaucratic obstacles (Greer et al., 1999), or simply free up time to focus on their core competencies. Often, the client merely gives directions to the provider by sharing the characteristics of good solutions, and allows the provider to search autonomously. For example, when Tesla delegated the design of Falcon wing doors to Hoerbiger, Tesla stipulated speed and symmetry tolerances that any design should meet and allowed Hoerbiger considerable flexibility (Irwin, 2016).

While giving autonomy to a provider allows the client to get access to a broader set of solutions, it simultaneously creates a significant challenge for the client by reducing her ability to control the flow of the search process.[1] In particular, this autonomous delegation limits the client's observability on search efforts, which can potentially lead to strategic effort exertion by the provider. Subsequently, due to such decentralization, the client's desire to find the best-possible solution may become misaligned with the provider's objective to find something satisfactory. Moreover, because search for the best alternative is typically not a one-shot affair and occurs over multiple iterations or rounds, once a viable solution is discovered, the provider may not share the client's enthusiasm to search for an even better solution. The client can influence the provider's efforts over time by managing structural or financial elements of the delegated search process. While prior research on the topic has focused on financial mechanisms to motivate the provider (for example, Terwiesch and Loch 2004; Kwon et al. 2010; Wu et al. 2014; Zhang 2016), we explore important structural search factors that are often used by clients in practice. For instance, the client could communicate her willingness to terminate the project by treating any acceptable solution as a breakthrough. Alternatively, the client can limit the strategic behavior of the provider by simply limiting the transparency of the search process. In other words, the client may not share information about the quality of any solution with the provider (if possible) unless and until it is implemented.

The primary objective of this paper is to understand the strategic interactions that occur between a

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[1] Throughout the paper, we refer to the client as "she" and to the provider as "he".

client and a provider in a delegated search process. Many factors such as the difficulty of the problem, the provider's capability in searching for a solution, and financial incentives in place, can influence the provider's and client's decisions in the search process. Several interesting questions arise in such contexts about the behavior of the participating firms, and the outcomes desired and obtained by them. To address these questions, outlined in §1.2 below, we build a model where the client and the provider interact sequentially over time to identify a solution to the client's problem. We characterize the client's and provider's optimal strategic behavior under various search structures and develop insights regarding how they depend on the problem and firm characteristics. In addition, we discuss how those insights can be applied in practice.

1.1   Practical Motivation[2]

Delegated Search in Creative Advertising. As an archetype of the delegated search process, consider the illustrative example of an Indian insurance firm, whom we call Chimera. In order to enter a new region of the country, Chimera worked with a multinational advertising firm called Fido to create a television commercial. Sales of Chimera's insurance policies were expected to increase in proportion to the number of consumers who will include Chimera in their consideration set; a commercial that helps a consumer remember Chimera would greatly facilitate a good sales season. Before the project commenced, it was also determined that a successful commercial would imprint Chimera's brand name in at least 60% of the viewers. A concept generated by Fido would be adopted and produced only if it achieved this performance level during the testing phase; however, Chimera may seek more solutions even after Fido delivered an acceptable solution. Delivering a creative solution even with well-defined objectives is not an easy matter in this context; even the best efforts of Fido could result in failures for Chimera. Fido's creative team did not possess the contextual depth of the client, which meant every concept had to be evaluated by a separate team from Chimera. However, the entire project would have to work under a strict time-line imposed by the television networks' sales windows for commercials (in this case, the firms had about 10 weeks to search for a solution). It was common knowledge that the sooner a concept was adopted, the better the opportunity to produce it and optimize its placement in various advertisement slots. Fido's initial attempts to develop a solution fell short, which caused alarm due to the impending deadline. Eventually, Fido inserted more creative talent into the team and produced a commercial that was cleared for production after testing. In this particular instance, the search was concluded after two iterations.

Delegated Search in Product Design. Similar issues arise in other contexts such as Product Development. Beta, an American Product Design Consulting firm, has extensive expertise in a variety of areas such as ethnography, product design, interaction design, etc. To illustrate the issues that motivate the paper,

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[2]The names of the providers and clients in these examples and some sensitive numbers have been disguised by request.

consider their engagement with Rialto, a home appliance manufacturer. Rialto hired Beta to generate product designs to enter a particular lucrative countertop appliance category. Rialto had determined that, based on various supply chain, costing and competitive parameters, the appliance would be profitable only if it can be produced at a margin of $17, or more. Beta would do the ground-level research to generate insights, and share a design with Rialto (typically including a 3-dimensional diagram, a bill of materials, etc.) . Rialto would take the design, get input from manufacturing and engineering stakeholders to determine the actual cost of producing a product based on the submitted design. As it was well known that the main selling season for this appliance category was during the winter holidays, Rialto had a target date by which any finished product would have to be placed on retail shelves. While any design that delivered a margin of $17 per unit would be acceptable, Rialto's goal was to produce the design with the highest margin. Therefore, Rialto did not commit that the search would end immediately if a design met the target.

1.2 Managerial Issues and Insights

The clients and providers in the examples confronted similar problems, albeit in very different settings. Consider delegated search in the advertising industry, several points about which are worth noting. First, at the outset, both Fido and Chimera had the incentive to conclude the search process in as few iterations as possible. For Fido, each iteration represented direct costs in terms of locked up creative talent and production expenses. For Chimera, each iteration involved an interruption of normal activities of the firm to evaluate the new commercial design proposed by Fido. Second, the project had a clear and explicit deadline by which a commercial had to be produced. If this deadline passed, Chimera would have been unable to optimally purchase slots in regional TV channels in time to enter the consideration set for the pre-tax personal insurance market. Third, Fido was able to adjust the intensity of the search effort based on the proximity of the deadline by changing the size and constitution of the creative team. Elevating the level of effort, however, did not guarantee that the search would yield a satisfactory commercial concept. To compound this, producing a satisfactory concept sooner did not guarantee that the client would put an end to the search (this is a significant departure from papers that focus on breakthrough projects).

The potent combination of these factors – the approaching deadline, the uncertainty of the search, and the opportunity for strategic behavior by both firms – raise several questions of managerial significance. In this paper, we address the following research questions that are relevant in such contexts: How do delegated searchers, or providers, allocate their search effort over time? How does this search effort depend on the structure of the search process? How does the client decide whether to continue the search or terminate it? Specifically, when an acceptable solution is found, should the client retain the flexibility to search more, or

should she commit to terminate the search (*as if* the solution is a breakthrough)? In order to improve the quality of the search process, should the client be transparent about the qualities of the provider's solutions, or keep them hidden (when possible) from the provider?

We develop a stylized model to answer these important questions. The client seeks a solution over two rounds, at the end of which an acceptable solution must be implemented (if it is found). The criteria for acceptability are known in advance and communicated to the provider. Each round of search yields the provider one solution, which he submits to the client for evaluation. The outcome − that is, the solution's quality − is stochastic and depends on (i) the difficulty of the problem, (ii) the capability of the provider, and most importantly, (iii) the provider's effort intensity. The client's profits are proportional to the quality of the implemented solution. The provider is paid a fee for each round of search and a fixed reward when a solution he provided is implemented. We study the provider's search efforts and client's decisions to seek further solutions over time under three distinct delegation structures: (C) Committed structure, where the client defines any acceptable solution as a terminal breakthrough; (O) Open-ended structure, where the client reserves the right to continue search even after receiving an acceptable solution, but gives feedback to the provider about past solutions; and (S) Silent Open-ended structure, where the client reserves the right to continue search even after receiving an acceptable solution, but does not give feedback to the provider about past solutions. These structures vary significantly in two important dimensions: the degree of flexibility the client has in continuing the search, and the information asymmetry the client can impose on the provider.

Our analysis of the provider's behavior and client's strategy yields the following insights. We show that the provider's behavior is more nuanced, and that it depends critically on the transparency and flexibility of the search process. In line with prior research (Zhang, 2016; Rahmani et al., 2016), we find that the provider has an incentive to procrastinate (by increasing his search effort over time) under all three structures. However, the level of procrastination can be worse in the Open-ended structure, which is fueled by the difficulty of pleasing the client in the early part of the search. In contrast, the Silent Open-ended search could minimize the provider's procrastination, as the lack of transparency induces the provider to smooth his efforts.

Our analysis of the client's expected profit yields interesting insights as well. For a long list of well-documented reasons, it is generally considered valuable for a firm to retain or create as much flexibility as possible in the way they operate (Cachon and Terwiesch, 2012; van Mieghem, 2008). Flexibility, in the context of delegated search, refers to the client's ability to seek more solutions even after an acceptable solution has been delivered by the provider. We derive conditions under which such flexibility is not only useless, but even counter-productive. In particular, we show that when working with a highly skilled provider, a flexible approach would curb the provider's incentive to exert high effort during the early stages of the search, thereby unduly delaying the conclusion of the search. Our comparison of transparency in feedback

reveals yet another valuable insight for clients. Our results show that for complex projects with more capable providers, the client would maximize her profit by committing to remain silent till a solution is ready to be implemented. This gives the provider an incentive to exert greater effort sooner (i.e., balancing his efforts over time) so that he can compel the client to end the search earlier. These results are discussed in greater detail and summarized in §4 and §5.

The rest of the paper is organized as follows. We review the related literature in the next section. We present our model for the three search structures in §3. We then compare the effort provision and efficiency of the three search systems in §4 and §5. Section 6 presents our conclusions. All proofs and technical details are gathered in Appendix.

2   Literature Review

The challenges faced by organizations as they search for solutions to new problems have been extensively documented and studied in many contexts (Kornish and Hutchison-Krupat, 2016). Simon (1969) discussed the difficulty of identifying a truly optimal solution in novel contexts, where a vast number of possible solutions is available. Even when the number of options is limited, sequencing their exploration is no trivial task (Weitzman, 1979). The optimal manner of searching and the efficiency of the search process is determined by the cost of exploration and the value of information (McCardle, 1985), and the deadlines faced by the organization (Kornish and Keeney, 2008). Many organizations facing a combination of these issues − such as the clients that motivate this paper − simply do not have the internal capability to search for solutions efficiently and therefore delegate the search to external providers who possess the expertise to generate solutions.

One key challenge for clients in delegating their search process is to manage the misalignment of their objectives with those of their solution providers. That is, due to this delegation of search, providers sometimes choose actions which are not necessarily desirable to their clients (Holmström 1982). One way to tackle this inefficiency is via designing monetary incentives. There exists a large literature on incentives design for delegated search processes with the purpose of mitigating inefficiencies due to misaligned actions (e.g., Kwon et al. 2010; Lewis 2012; Manso 2011; Wu et al. 2014) or information (e.g., Terwiesch and Loch 2004; Ulbricht 2016). However, in the real world situations under our consideration, the monetary incentives are often determined by factors such as industry dynamics, the provider's reputation, and the nature of the client's problem, and thus the client may not have direct control on them. We therefore intend to study other sources of incentives beyond monetary transfers that can be managed by clients.

Horner and Skrzypacz (2016) review existing papers that consider other channels of incentives in dynamic

delegated search processes, particularly via the principal's information disclosure and control on stopping the process. According to Horner and Skrzypacz (2016), the majority of papers on delegated search processes are on situations where the client is searching to learn about her exiting alternatives, using a two-armed bandit model (e.g., Bolton and Harris 1999; Keller et al. 2005). While those studies generate valuable insights for managing search among alternatives, our focus in this paper is on a different type of search process. That is, we consider a situation where the client is looking for new and creative solutions. We therefore do not restrict the solution space into exiting alternatives. Similarly, Horner and Skrzypacz (2016) report that "restriction to two independent arms [..] is probably not the best way to think of a process of discovery and innovation." Accordingly, instead of limiting search among exiting alternatives with unknown properties, we assume that the quality of solutions can vary depending on the intensity of the provider's effort, which is unobservable to the client. For instance, in creative projects such as new product design or advertising, the client often does not have access to provider's backstage effort. Instead, she has a better ability to evaluate the suitability of the solution to her particular problem. This modeling approach allows us to study two kind of non-monetary approaches that the client can pursue to motivate the provider to exert costly effort on search, namely via committing to a breakthrough termination and via providing feedback. We next discuss the related literature along these two dimensions.

In delegated search processes, clients can commit to a breakthrough search ex ante (Bonatti and Horner 2011, Zhang 2016, Rahmani et al. 2016). In such situation, the search is terminated as soon as the provider offers a solution that its quality exceeds the breakthrough threshold. The existing studies on breakthrough (or committed) search processes have captured situations were only the provider (or agents) can exhibit strategic behavior with the focus on mitigating the search inefficiencies via monitoring (Bonatti and Horner 2011), setting deadlines (Zhang 2016), and leadership approach (Rahmani et al. 2016). We complement those studies by considering a situation where the client can also exhibit strategic behavior by not committing to a breakthrough. That is, unlike the breakthrough models where achieving a certain goal guarantees the conclusion of the search process, we consider situations where the client can choose to continue the search even after an acceptable solution is offered by the provider. This "open-ended" search approach by the client is a noticeable distinction because, in that case, the provider's choice of effort would be based on his anticipation about the client's stopping threshold as opposed to an exogenously specified threshold. Our results generate insights as how the client's commitment can impact the provider's efforts as well as the search efficiency.

The availability of the open-ended approach makes the client, in this study, more strategic than in prior papers. Furthermore, providing feedback to the provider (i.e., client being transparent about the quality of the previously submitted solutions) assumes an important role in this setting. Since the existing literature

largely focuses on breakthrough models of search, providing feedback to a provider is not critical in those studies. An exception to this is Manso (2011), who considers an agent who chooses between risky and safe approaches to generate solutions, but does not choose the level of effort to be exerted. Manso considers a situation where only the client can observe the outcome of the risky arm, and then, she can choose whether to share that information with the provider or not. He shows that providing feedback is essential for motivating workers to choose the risky option (obviously, when that is desirable for the client). In contrast, we consider providers who consciously choose their effort level at each stage, and find that providing feedback can be counterproductive as it can elevate the degree of procrastination.

A different context in which search and the role of feedback have been studied is open innovation, where firms have used emerging platforms to delegate the search to a community of agents (Chesbrough, 2003). Firms have used this open model of innovation effectively when solution diversity is valuable (Terwiesch and Xu, 2008; Erat and Krishnan, 2012), problem solving skill is distributed and not transparent (Boudreau et al., 2011), or it is efficient to evaluate and process a multitude of solutions (e.g., Kornish and Ulrich, 2014). This literature has focused on the role of feedback in sequential ideation, where feedback is provided to participants about their status compared to others (Bimpikis et al., 2016; Bockstedt et al., 2016; Mihm and Schlapp, 2016; Wooten and Ulrich, 2016). While these studies generate valuable insights on managing open innovation processes, as aforementioned above, the focus of this paper is on dedicated search processes, where the search is delegated to an agent or a team who is specialized in that as in our motivating examples in §1.1. Accordingly, in the next section, we develop a model of bilateral search engagement where feedback can be provided about the quality of previous solutions.

3   Model

In this section, we model the repeated interaction in a bilateral supply chain of *ideas*. The buyer of ideas, whom we refer to as the *Client*, is interested in seeking an acceptable solution for a business problem. The solutions are generated and supplied to the client by a *Provider*.

3.1   The Solution Space: Generating and Evaluating Solutions

To parsimoniously capture the dynamic nature of the interactions between the provider and client, we consider a stylized setting in which the client must conclude the search by a deadline; in particular, the client should adopt a solution within two rounds. The existence of a deadline is a reality in nearly all delegated search scenarios (Gersick 1988, 1989; Eisenhardt et al. 1995; Lindkvist et al. 1998; Kerzner 2013); indeed, some prior research on this topic has implicitly assumed that the deadline is strictly one round (e.g.,

Erat and Krishnan, 2012).

The client's profit increases with the quality of the solution she is able to implement. For example, an appliance manufacturer's profit margins are higher if the cost of the implemented appliance design is lower. For simplicity, we assume that the client's profit is the same as the implemented solution's quality. However, the client will not be able to implement a solution if it fails to meet certain requirements, which we describe in greater detail below.

Acceptable Solutions: The client is not capable of internally generating solutions to the problem she faces, and relies on the provider to generate solutions. At the outset, the client communicates to the provider the nature of her problem, which will help establish the difficulty of finding solutions that meet the client's needs. Instances of such interactions are commonplace: a home buyer will indicate parameters of a new house she is looking for to a real estate agent; a website's manager may share performance details of the existing recommender system and mention the level of improvement she seeks; a hospital administrator would clarify to a human resources firm the minimum experience level and desired characteristics of the specialist they seek to hire; or, an appliance manufacturer could identify for a professional design firm the highest production cost that can be supported in a new segment for whole-fruit juicers. As suggested by examples such as these, the provider gets an understanding of the minimal performance level of a solution that the client would find *Acceptable*, which we refer to as A; we assume this to be a uni-dimensional factor with $A \geq 0$.

In practice, the acceptability level A could be a confluence of several factors. These could be (i) the fixed cost the client would incur in implementing any solution, (such as the product launch costs incurred after a design is accepted), (ii) the performance level of an incumbent solution, or more generally, a reference point (such as the accuracy of an existing recommender system at an internet retailer), (iii) a technical, institutional or personal requirement (this could be a custom-designed chip's thermal packaging, a hospital's requirement to obtain a Harvard-educated neurosurgeon, or a very particular home buyer's configuration preferences).

All in all, the acceptability level is an indicator of the provider's "difficulty" in meeting the client's expectations. If the level of the solution does not meet or exceed the level A, the client will not benefit by accepting the solution. However, providing an acceptable solution does not ensure that the client will adopt it immediately; we discuss this in detail later.

Generating Solutions: In each round, the provider can generate one solution, whose "quality" is uncertain. The quality of each outcome is stochastic and depends on (i) the difficulty or acceptability level of the problem A, (ii) the capability of the provider k, and most importantly, (iii) the provider's effort intensity μ. That is, the provider can exert effort to improve the distribution of outcomes. For instance, in a professional design firm, the provider can change the structure of his team by adding (or removing) highly skilled designers over

time to generate solutions that are more (or less) suitable for the client. In the hiring context, the recruiting firm can exert low effort by merely posting the job on their website and collecting the submitted curriculum vitae; alternatively, they can exert additional effort and contact people who might fit for position better, even if they are employed elsewhere.

The provider's efforts are private, and cannot be observed or verified directly by the client. While the client cannot verify the exact effort exerted by the provider, she can verify that whether a solution is generated or not. Thus, the provider should commit to a minimum effort level required for generating a solution in each round. We denote that minimum effort committed by the provider by μ_0. This minimum effort μ_0 can also be interpreted as a "due diligence" effort that must be exerted by the provider before a solution can be submitted; for example, a design firm should ensure that the generated design does not violate design patents held by other firms. Accordingly, we define the provider's choice of effort in round t by $\mu_t \in [\mu_0, 1]$. Similar to Kavadias and Sommer (2009), we assume that the quality of solutions follows an exponential distribution. The exponential distribution fits our search context because it does not bound from above the quality of the solution. That is, when the provider exerts effort of μ_t in round t, the probability of obtaining a solution with a quality of at least A, in that round, is given by $\bar{\Phi}_t(v) = \mu_t e^{-A/k}$; the expected quality of the solution is $k \cdot \mu_t$, which is increasing in the provider's effort (μ) and his capability (k). In addition, the success rate is decreasing in the acceptability level A.

The provider's cost in each round increases with his effort μ_t; we assume that the cost of effort is convex increasing and for simplicity consider that as being quadratic, i.e. $c(\mu) = c\mu^2$ with $c \geq 0$. Such a cost function has been widely used in the literature (e.g., Bhattacharyya and Lafontaine, 1995; Bhaskaran and Krishnan, 2009; Kwon et al., 2010), and is shown to be consistent with the empirical evidence reported by Cohen and Klepper (1992), and Kamien and Schwartz (1992).

Evaluating Solutions: A solution's quality can be assessed only after the provider presents the solution to the client. For instance, consider the example of Rialto, the appliance manufacturer who delegates designs to Beta. The profitability of any design will depend on the production cost of the implemented design, which can be estimated only after the design is reviewed by Rialto's production team. The client incurs a fixed cost of c_I for each round of idea evaluation. Clients may incur these costs for a combination of reasons including the direct cost of idea evaluation (such as prototyping and destructive testing), administrative costs incurred to organize meetings, cost of communicating with the provider, or the indirect cost associated with delaying an action plan.

In summary, with each round of activity, additional costs are incurred by both parties. For the client, a new solution comes with the cost of evaluation c_I; and for the provider, generating a solution adds a cost of $c(\mu)$. Therefore, both parties will benefit from a faster and fruitful conclusion of the search process. The

desirability of such an outcome is directly observable in practice. For example, in the context of Fido and Chimera from our motivating example, both the advertising firm and the insurance company would prefer to find the ideal commercial as soon as possible. Similarly, both a product design firm and its client will, ceteris paribus, prefer an earlier conclusion of the search process.

3.2   Search Structure and Incentives

Search Structure: At the outset, the client determines the structure of the search and communicates it to the provider. The client makes this choice along two important dimensions: First, the client determines whether the project will search for a *breakthrough*, or reserve the right to continue the search even after an acceptable solution is found. This search structure without a breakthrough commitment, while quite common in the real world, has not received sufficient attention in the literature, especially in a bilateral context. On the second dimension, the client decides whether to give *feedback* to the provider about the evaluation result of the previous solution (regarding whether it was acceptable or not). We assume that if the client chooses to not give feedback, the provider is unable to observe the quality of the solutions. However, he will be able to observe if the client implements one of those solutions. In many fields such as design, architecture, script-writing, and advertising, this observability of implementation curbs the client's ability to appropriate a solution without rewarding the provider (Stim 2011).

We consider the following three different delegated search structures that span the breadth of possibilities for the client along the aforementioned dimensions:

- *Committed*: The client commits to treat the engagement as the search for a breakthrough solution. If any of the provider's solution, upon evaluation, achieves or exceeds the acceptable threshold of A, the client immediately terminates the search and implements the solution. Note that under this structure, the choice of providing or not providing feedback is not applicable, because the client implements the first solution that achieves A.

- *Open-ended*: The client reserves the right to continue the search process even after an acceptable solution is found, but commits to provide feedback to the provider upon evaluating each solution (regarding whether it achieved A or not). That is, the client may ask the provider to search for another solution although an acceptable solution is found. This flexibility in the search process could be desirable since it allows the client to search beyond the first breakthrough.

- *Silent Open-ended*: In this structure, the client neither commits to a breakthrough termination, nor to give any feedback about any of the provider's solutions. Indeed, the client does not give feedback
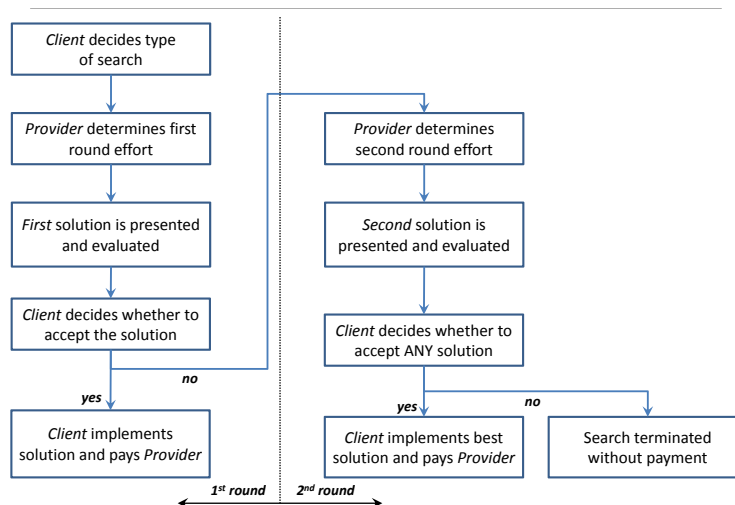
Figure 3.1: Timeline of Decisions and Events

to the provider upon evaluating each solution, unless she terminates the search. In addition, the client still reserves the right to continue the search process even after an acceptable solution is found. That is, the client may ask the provider to search for another solution without revealing any information about whether submitted solutions were acceptable or not. Clients might find this information asymmetry advantageous in motivating the provider in the second round.

Payments: The provider receives payments in two parts: (i) $\mathbf{w}$: a fee (or wage) that the client pays the provider for each round in which the provider's search services are employed; (ii) $\mathbf{F}$: the total fixed fee paid to the provider for finding a solution that will be implemented by the client. This payment scheme is a stylized version of fixed-fee and time-and-materials contracts, which are the most common in delegating search processes (Bartrick 2013). Each of these fees are known in advance, and are determined by factors such as industry dynamics, the provider's reputation, and the nature of the client's problem. We restrict ourselves to cases where $A > F$ in order to focus on meaningful searches where the client has an incentive to implement an acceptable solution.[3]

Recall that the client receives no reward if no acceptable solution is found during the search process. If at least one of the generated solutions is acceptable, the client's profit is the *best* among all of those solutions. At the same time, the client would prefer the search to be efficient. Therefore, as we will see in the rest of the paper, the client's choice of structure and subsequent behavior are governed by a trade-off between

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[3] It is possible that the provider receives a portion of the reward F as soon as an acceptable solution is found (say $\alpha \in (0,1)$), and the rest of the reward after a solution is implemented. The Committed and Open-ended search structures are alternative extremes of this general contract with $\alpha = 1$ and $\alpha = 0$, respectively. We show in Lemma 9 in the appendix that limiting our attention to Committed and Open-ended search processes is without loss of generality as these two types of search generate the same variety of behaviors as with more general contracts. Note that under the Silent Open-ended search, only the case with $\alpha = 0$ is applicable, because the client does not reveal any information about whether submitted solutions were acceptable or not.
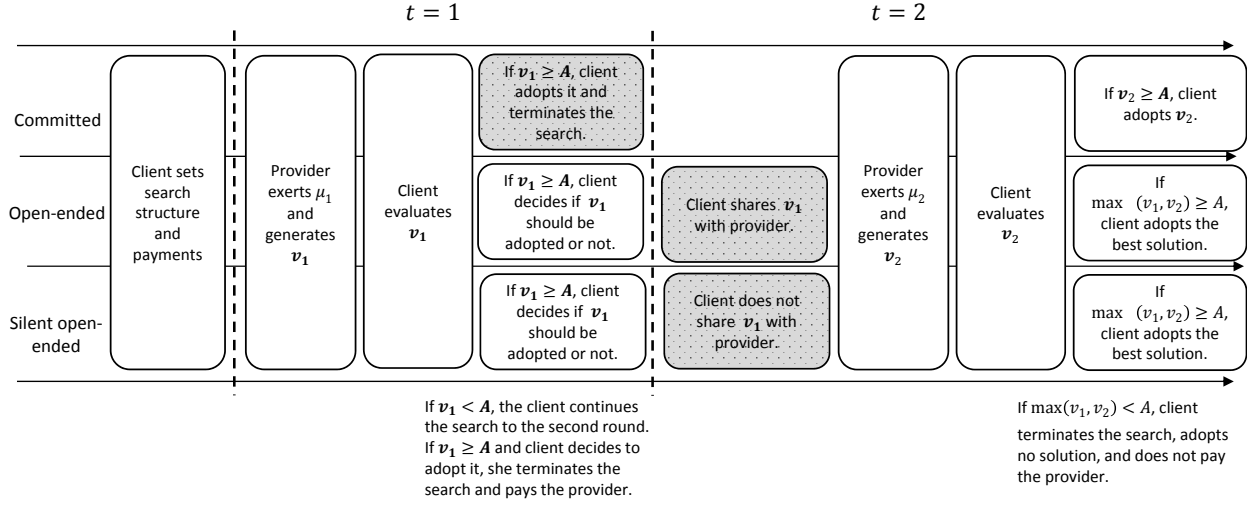
Figure 3.2: Decision space and payments of the three structures

these two forces. Further, to focus on the longitudinal elements of strategic interactions, we assume that the search structure, the costs of two parties, difficulty of the problem, and the capability of the provider are common knowledge. The sequence of events and decisions are provided in Figure 3.1. The decision-space for the three search structures and the attendant payoffs are shown in Figure 3.2.

4 Equilibrium Choices

In this section, we characterize the provider's equilibrium efforts and the client's stopping decisions under the three search structures. To simplify the exposition, we assume that the provider's unit cost of effort c is large enough that his efforts, in equilibrium, never reach the upper-bound, i.e., $\mu_t < 1$.[4] In addition, to avoid trivial cases, we assume that the payment F is set such that both the provider and client participate in the search process, and continue the search to the second round if the first solution is not acceptable. See details in Lemmas 2 and 3 in appendix.

4.1 Committed Search

In Committed search process, the client does not exhibit strategic behavior as she commits to treat the search as a a breakthrough solution contingent on achieving A. However, the provider can exhibit strategic behavior by adjusting her effort in each round, which determines the quality of solutions (probabilistically). We therefore model the provider's problem as a stochastic dynamic programming, in which he chooses his

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[4] Accounting for those boundary cases leads to similar insights at the expense of more complicated exposition.

efforts in each round by maximizing his individual expected payoff-to-go.

We denote by μ_t^C the provider's equilibrium effort under the Committed search in round t for $t \in \{1, 2\}$. The provider first determines the effort level for the first round (μ_1^C) and generates a solution for which he receives a fee w. This solution is then evaluated by the client, which determines whether the breakthrough is achieved or not. Let the quality of the first solution be v_1. The client terminates the search and implements the solution if $v_1 \geq A$. At this point, a payment of F is made to the provider for his successful search. If $v_1 < A$, the client will continue the search into the second round by paying another search fee of w to the provider. The provider then chooses effort level μ_2^C for the second round, armed with the knowledge that the solution generated in the first round was short of the acceptable level. The decision-space for the Committed search and the attendant payoffs are shown in Figure 3.2.

In the next proposition, using backward induction, we characterize the closed-form solutions for the provider's choice of effort in each round of the Committed search.

Proposition 1. *Under the Committed search, the provider's efforts in each round are as follows:* $\mu_1^C = \max\left\{\mu_0, \frac{\left(c \cdot (\mu_2^C)^2 - w + F \cdot \left(1 - \mu_2^C e^{-A/k}\right)\right) \cdot e^{-A/k}}{2c}\right\}$ *and* $\mu_2^C = \max\left\{\mu_0, \frac{F \cdot e^{-A/k}}{2c}\right\}$. *In addition,* $\mu_2^C \geq \mu_1^C$.

Proposition 1 shows that under the Committed search, the provider chooses to procrastinate; that is, he exerts lower effort in the first round and increases his effort in the second round. The degree of the provider's procrastination depends on the acceptability level A. In the next proposition, we study how the provider's effort provision depends on A.

Proposition 2. *(i)* μ_2^C *is decreasing in* A. *(ii)* μ_1^C *is uni-modal in* A; *specifically, there exists a threshold* $A^C \doteq \frac{k}{2} \cdot \ln\left(\frac{3F^2}{4c(F-w)}\right)$ *such that* μ_1^C *is increasing in* A *for* $A \leq A^C$ *and it is decreasing in* A *otherwise.*

Figure 4.1 illustrates our results in Proposition 2. Proposition 2 (i) shows that the provider's efforts in the second round is decreasing in A. This happens because a tough target (i.e, high A) discourages the provider to exert costly effort. However, Proposition 2 (ii) shows that the provider's effort in the first round is unimodal in A. This non-monotonicity of the provider's effort is due to two opposing forces, namely the provider's productive desire to succeed quickly (i.e., to reach A and receive his fixed fee F sooner and thus save the cost of generating a second solution), and his negative desire to delay the search (in order to receive a w for the second round of search, especially when w is large).

Particularly, in Figure 4.1, we can consider three regions: (I) when the target is soft (A is small), the provider chooses to exert low effort in the first round and high effort in the second round. In this case, because the likelihood of success is very high, it is optimal for the provider to procrastinate and delay the project so that he can save in first round costs and also collect his wage in both rounds. Note that because
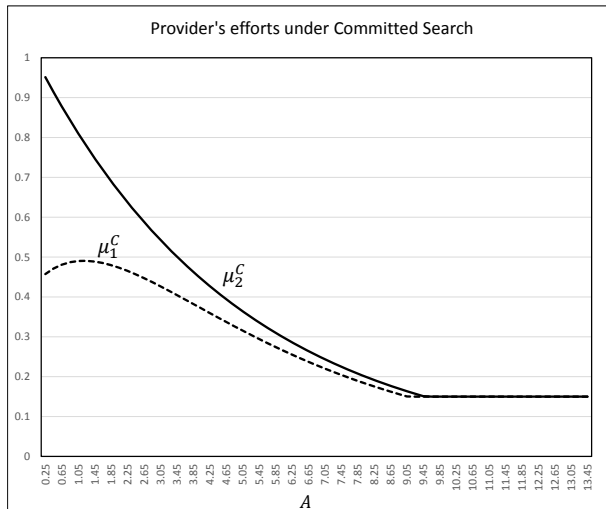
Figure 4.1: Provider's effort provision under Committed search
Parameters: $F = 3$, $w = 0.2$, $c = 1.5$, $k = 5$, $\mu_0 = 0.15$, and $c_I = 0.1$.

the provider's effort is not verifiable by the client, he receives his wage in each round irrespective of his effort level. (II) when the target is moderately tough (A is intermediate), the provider chooses to balance his efforts among the two rounds (i.e., increase his effort in the first round and decrease his effort in the second round.) In this case, because the likelihood of success is moderate, it is optimal for the provider to increase his overall chances of success by exerting relatively high efforts in both rounds. Finally, (III) when the target is tough (A is large), the provider chooses to exert low effort in both rounds. In this case, because the chance of success is low, the provider chooses to reduce his efforts in both rounds to save in costs and also collect his wage w twice.

4.2 Open-ended Search

In Open-ended search process, both the client and the provider can exhibit strategic behaviors. In particular, the provider chooses how much effort to exert and the client chooses whether to stop or continue the search even when the outcome of the first round is acceptable. We therefore model the problem as a stochastic dynamic game between the client and the provider, in which they choose their actions sequentially and by maximizing their individual expected payoffs to-go.

In this search process, the provider first determines his effort level for the first round, denoted by μ_1^O, and generates a solution for which he receives a fee w. This solution is then evaluated by the client, who then decides to continue or terminate the search. Let the quality of the first solution be v_1. If $v_1 \geq A$, the client may choose to adopt the solution or extend the search to the second round, while she gives feedback to the provider regarding that whether the first solution has been acceptable or not. Thus, the provider's

equilibrium effort in the second round under the Open-ended search depends on the value of the solution generated in the first round (v_1). Consequently, we denote by $\left(\mu_2^{O-}, \mu_2^{O+}\right)$ the provider's equilibrium effort under the Open-ended search in the second round when $v_1 < A$ and when $v_1 \geq A$, respectively.

If $v_1 < A$, the client will continue the search into the second round by paying another search fee of w to the provider, and the provider exerts effort μ_2^{O-}. If $v_1 \geq A$ and the client chooses to extend the search, the provider will receive the search fee w, and in return exert effort μ_2^{O+} to produce a new solution (of quality, say, v_2). If $v_2 > v_1$, the client will adopt the new solution; otherwise, the client will implement the solution from the first round (as it was acceptable). It must be noted that, when $v_1 \geq A$, regardless of the outcome in the second round, the provider will receive his fixed reward of F at the end of the second round. The decision-space for the Open-ended search and the attendant payoffs are shown in Figure 3.2. In the next proposition, we characterize the client's stopping threshold, which determines when the client adopts the solution of the first round or extends the search to the second round.

Proposition 3. *Under the Open-ended search, there exists a threshold $v_s^O \doteq k \cdot \ln\left(\frac{\mu_0 k}{c_I + w}\right)$ such that it is optimal for the client to stop the search and adopt the first solution (v_1) if and only if $v_1 \geq \max\left\{A, v_s^O\right\}$.*

Proposition 3 shows that the client stops the search if the outcome of the first round is higher than a threshold. Note that if $v_s^O \leq A$, the client stops the search for any outcome that satisfies A. Hence, in that situation, the Open-ended and Committed search processes perform the same. However, when $v_s^O > A$, the Open-ended and Committed search processes perform differently, because for $A \leq v_1 < v_s^O$, the client chooses to continue the search under the Open-ended search, but not under the Committed search.

In addition, it can be shown that v_s^O is increasing in the provider's productivity factor k and the minimum committed effort μ_0, because the higher these parameters the more likely that the search in the second round results in a better outcome. In contrast, the threshold is decreasing in the client's inspection cost c_I and the provider's wage w, because the client's payoff is decreasing in these parameters. When the client's evaluation cost is high, it is optimal for the client to set the bar low and stop the search even when the first solution is just slightly higher than A. On the other hand, when her evaluation cost is relatively low, it is optimal for her to set the bar high and stop the search only if the outcome of the first round is considerably higher than A.

In the next proposition, using backward induction, we characterize the closed-form solutions for the provider's choice of effort in each round of the Open-ended search. Note that the provider chooses his effort levels by anticipating the client's stopping threshold characterized in Proposition 3.

Proposition 4. *Under the Open-ended search, the provider's efforts in each round are as follows: $\mu_1^O =$*
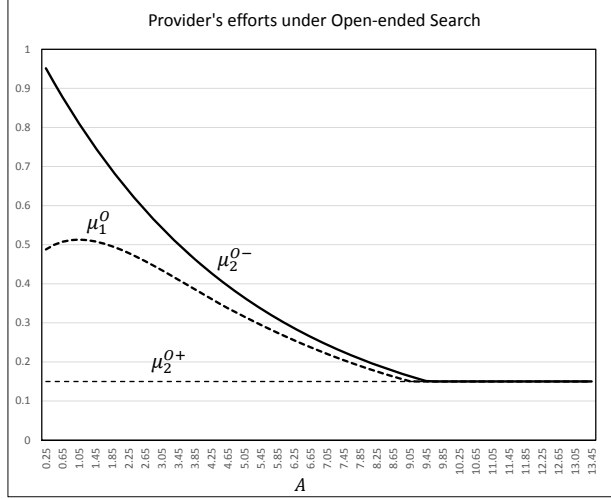
Figure 4.2: Provider's effort provision under Open-ended search
Parameters: The same as in Figure 4.1.

$$\max\left\{\mu_0, \frac{\left(e^{-A/k}-e^{-\max\left\{A,v_s^O\right\}/k}\right)\cdot\left(-c\mu_0^2+w\right)+\left(c\cdot\left(\mu_2^{O-}\right)^2-w+F\cdot\left(1-\mu_2^{O-}e^{-A/k}\right)\right)\cdot e^{-A/k}}{2c}\right\}, \mu_2^{O-}=\max\left\{\mu_0, \frac{F\cdot e^{-A/k}}{2c}\right\},$$

and $\mu_2^{O+}=\mu_0$. In addition, $\mu_2^{O+}\leq\mu_1^O\leq\mu_2^{O-}$.

Proposition 4 shows that under the Open-ended search, the provider's effort may increase or decrease over time depending on the outcome of the first round. In particular, if the outcome of the first round does not hit the client's acceptable threshold A, it is optimal for the provider to increase his effort in the second round; otherwise, it is optimal for him to decrease his effort. The reason for this change of approach is as follows: On one hand, when the outcome of the first round is not acceptable, the provider has only one more chance to succeed and collect his reward F. Thus, it is optimal for him to enhance the likelihood of success in the second round by exerting more effort. On the other hand, when the outcome of the first round is acceptable, although the client is looking for a better solution, the provider does not share the same enthusiasm to search for a better solution. That is, because in this situation, the outcome of the first round has been acceptable, the provider's payment is guaranteed irrespective of the outcome of the second round. Thus, it is optimal for him to only exert his minimum committed effort. In the next proposition, we study how the provider's effort provision depends on the acceptability level A.

Proposition 5. *(i)* μ_2^{O-} *is decreasing in A. (ii)* μ_2^{O+} *is independent of A. (iii)* μ_1^O *is uni-modal in A; specifically, there exists a threshold $A^O \doteq \frac{k}{2}\cdot\ln\left(\frac{3F^2}{4c\left(c\mu_0^2-F\right)}\right)$ such that μ_1^O is increasing in A for $A\leq A^O$ and it is decreasing in A otherwise.*

Figure 4.2 illustrates our results in Proposition 5. Similar to the Committed search, the provider's efforts in the second round when $v_1 < A$ (i.e., μ_2^{O-}) is decreasing in A, because a tough target (i.e, high A)

discourages the provider to exert costly effort. However, when $v_1 \geq A$, the provider's effort in the second round is independent of A, because he does not have incentives to exert any effort above his minimum committed. Finally, similar to the Committed search, the provider's effort in the first round is unimodal in A. However, the intensity of his effort will be different from the Committed search. In Section 5, we compare the provider's efforts under the three search structures.

4.3   Silent Open-ended Search

The flow of the Silent search process is similar to the Open-ended search and is illustrated in Figure 3.2. The only difference is that under the Silent search, when the client chooses to extend the search to the second round, she does not give feedback to the provider that whether the first solution has been acceptable or not.

This information asymmetry has two impacts on the provider's payoff and subsequently his choice of effort: First, unlike the Open-ended search where the provider's effort in the second round is conditional on the value of the first solution, under the Silent case, the provider chooses his equilibrium effort in the second round based on his expectation about the value of the first solution, i.e., $\mathbb{E}_{\mu_1^S}[v_1]$. Second, in contrast to the Open-ended search where achieving $v_1 \geq A$ would guarantee the provider's reward F irrespective of the outcome of the second round, under the Silent case, the provider has no confidence about receiving his reward F, because information about v_1 is not revealed to him.

From the client's perspective, this information asymmetry can be beneficial in encouraging the provider to exert a higher effort in the second round. In particular, unlike the Open-ended search, where the provider only exerts his minimum committed effort when the solution of the first round is acceptable, under the Silent search, the provider may choose to exert a higher effort, because he does not know the value of the first solution and thus is less confident about receiving his reward F.

We denote by μ_t^S the provider's equilibrium effort under the Silent Open-ended search in round t for $t \in \{1, 2\}$. In the next proposition, we determine the provider's equilibrium effort in the second round if the client chooses to continue the search, and also characterize a threshold that determines when the client adopts the solution of the first round or extends the search to the second round. Note that the provider chooses his equilibrium effort in the second round based on his expectation about the value of the first solution. Thus, his effort in the second round (μ_2^S) depends on the effort he exerted in the first round (μ_1^S).

Proposition 6. *For a given μ_1^S, the provider's and client's actions in the second round are as follows:*

(i) If the client chooses to continue the search to the second round, the provider's effort is equal to $\mu_2^S = \max\left\{\mu_0, \frac{e^{-A/k}F}{2c} + \mu_1^S \cdot Z\right\}$, *in which* $Z \doteq \frac{(c_I + w)}{k} - \frac{\left(e^{-A/k}\right)^2 F}{2c} \leq 0$.

(ii) There exists a threshold $v_s^S\left(\mu_2^S\right) \doteq k \cdot \ln\left(\frac{\mu_2^S k}{c_I + w}\right)$ *such that it is optimal for the client to stop the*

search if and only if $v_1 \geq \max\left\{A, v_s^S\left(\mu_2^S\right)\right\}$.

Proposition 6 (i) shows that the provider's effort in the second round is linear and decreasing in his effort in the first round. This implies that the provider's efforts among the two rounds are strategic substitutes. That is, the provider may choose, in equilibrium, to reduce his second round effort by exerting a higher effort in the first round.

Proposition 6 (ii) shows that the client's stopping threshold $v_s^S\left(\mu_2^S\right)$ is increasing in the provider's effort in the second round μ_2^S. Naturally, the higher the effort of the provider in the second round, the more likely that the search in the second round results in a better outcome, and it would therefore become beneficial for the client to set a higher bar for the search process. However, because μ_2^S is decreasing in μ_1^S, the client's stopping threshold $v_s^S\left(\mu_2^S\right)$ is also weekly decreasing in the provider's effort in the first round μ_1^S. This is in contrast to the Open-ended search where the client's stopping threshold is independent of the provider's effort in the first round. Note that under the Open-ended search and when $v_1 \geq A$ is released, the provider's effort in the second round is constant and equal to his minimum committed effort, i.e., $\mu_2^{O+} = \mu_0$. However, under the Silent search, because v_1 is not released, the provider's effort in the second round can be higher than μ_0.

The above results imply that, from the provider's perspective and under the Silent search, there are two important benefits of exerting a higher effort in the first round: First, by increasing his effort in the first round, the client's stopping threshold decreases; that is, the client sets a lower bar for the termination of the search process. Thus, it is more likely that the search terminates at the end of the first round, which may particularly be desirable for the provider when F is large and w is relatively small. Second, unlike the Open-ended and Committed search, where the provider's efforts in the second round are independent of his efforts in the first round, under the Silent search, the provider can strategically substitute his effort in the second round with his effort in the first round. The benefit of such efforts substitution is that by exerting a higher effort in the first round, the provider will have a higher confidence about the his first solution hitting A, and thus he can strategically reduce his effort in the second round.

From the client's perspective, note that if $v_s^S\left(\mu_2^S\right) \leq A$, the Silent search resembles a breakthrough search as the client stops the search for any outcome that satisfies A. When $v_s^S\left(\mu_2^S\right) > A$ and $\mu_2^S = \mu_0$, although the client's stopping threshold will be the same under the Silent Open-ended and Open-ended search processes, the provider's equilibrium efforts still vary under the two structures due to information asymmetry. In Section 5, we compare provider's effort and performance of the three search structures.

In the next proposition, using backward induction, we characterize the provider's equilibrium efforts in the first round of the Silent search. Note that the provider chooses his effort levels by anticipating the effect
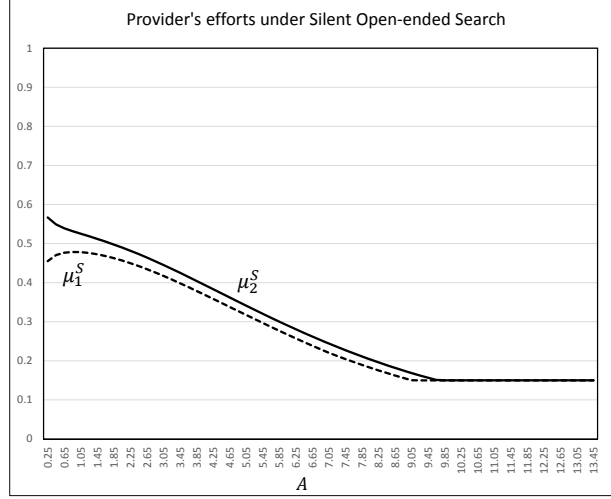
Figure 4.3: Provider's effort provision under Silent Open-ended search
Parameters: The same as in Figure 4.1.

of that on the client's stopping threshold and his second round effort, characterized in Proposition 6.

Proposition 7. *(i) Suppose* $\mu_2^S = \mu_0$. *Then,* $\mu_1^S = \max \left\{ \mu_0, \frac{e^{-A/k}F\left(1-\mu_0 e^{-A/k}\right)+\left(\frac{c_I+w}{\mu_0 k}\right)\left(c\mu_0^2-w\right)}{2c} \right\}$. *(ii)*
Suppose $\mu_2^S = \frac{e^{-A/k}}{2c}F + Z \cdot \mu_1$. *Then,* $\mu_1^S = \max\left\{\mu_0, \hat{\mu}_1\right\}$, *in which* $\hat{\mu}_1$ *is the unique solution to* $\Theta(\mu_1) = 0$
in the interval $[0, 1]$ *such that*

$$
\begin{aligned}
\Theta(\mu_1) &\doteq 2c \cdot \left(Z^2 + F \cdot e^{-\left(A+v_s^S(\mu_1)\right)/k}\left(\left(\mu_2^S(\mu_1)\right)^2 + w\right)\right) \cdot \mu_1 \\
&+ \left(Fe^{-A/k} - \mu_1 \cdot \left(\mu_2^S(\mu_1)\right)^{-1} \cdot e^{-v_s^S(\mu_1)}\left(\left(\mu_2^S(\mu_1)\right)^2 - w\right)\right) \cdot Z. \quad (4.1)
\end{aligned}
$$

Figure 4.3 illustrates our results in Proposition 7. It can be seen that the provider's efforts are more balanced under the Silent search compared to the Open-ended and Committed search processes illustrated in Figures 4.1 and 4.2, respectively. In particular, as we discussed after Proposition 6, knowing that the client will not reveal any information about the outcome of the first round, the provider is better-off increasing his effort in the first round so that he can increase the likelihood of hitting A in the first round, which has also two other advantages: it reduces (i) the client's stopping threshold and (ii) the provider's effort in the second round. This change in provider's effort, due to information asymmetry, reduces the degree of procrastination under the Silent search process. Thus, in contrast to the Committed and Open-end search (when $v_1 < A$) where the second solution has considerably higher expected value than the first one (due to higher degree of procrastination), under the Silent search, the expected value of the two solutions are more aligned.

In the next section, we compare the provider's and client's equilibrium choices, and the relative performance of the three search structures.

5 Additional Analysis and Discussions

In the previous sections, we analyzed the outcomes of the delegated search process under three different structures, which varied in the level of flexibility (for the client) and feedback given (by the client). In this section, we compare the three structures in two separate dimensions. First, we compare the equilibrium stopping thresholds of the client and the inter-temporal efforts of the provider. Secondly, by comparing the equilibrium expected value of the client's utility across the structures, we develop insights on the conditions under which each of these structures should be pursued by the client.

5.1 Comparison of Equilibrium Choices

Client Behavior. We first consider how the behavior of the client changes between the different structures. The client's primary influence during the delegated process is through her response to an acceptable solution. Whereas in Committed search the client immediately terminates the search upon receiving an acceptable solution, her decision is less obvious when the client has flexibility to continue following a breakthrough. From the provider's point of view, a client who seeks additional solutions in spite of receiving an acceptable solution might seem "difficult" or "challenging" (Chu, 2016; Holmes, 2015). However, our analysis shows that the difficulty of pleasing a client in the first solution is endogenous and sensitive to the structure of the search itself. To derive further insight, in Proposition 8 below, we compare the optimal stopping thresholds of the clients in the Open-ended structures.

Proposition 8. *i) The client's stopping threshold is higher under the Silent search than the Open-ended search, i.e., $v_s^S \geq v_s^O$.*
ii) Further, if $k > \bar{k}$, $v_s^S \geq v_s^O > A$, where $\bar{k}$ is the solution to the equality $\bar{k} \cdot \ln\left(\frac{\bar{k}\mu_0}{c_I + w}\right) = A$.

Proposition 8 reveals two important results about what makes clients harder to satisfy from the provider's perspective. First, since $v_s^S \geq v_s^O$, a client who is Silent and withholds feedback not only *seems* more difficult to please with a solution, but is *actually* harder to please. This is best explained by considering the client's objective when the first solution provided by the client already exceeds the threshold. In this scenario, the client will terminate the search only if the expected marginal improvement over the first solution exceeds the additional cost of search ($c_I + w$). When the client is transparent, the provider may find the client demanding, but exerts minimal effort in the second round (when $v_1 \geq A$) because he is aware of the quality of the first solution. On the other hand, if the client is Silent, the provider will exert more effort because he has no assurance about the acceptability of the first solution. Thus, because the likelihood of receiving a second solution of higher quality is higher under the Silent than open (only when the first solution is

acceptable), the client's expected payoff upon continuation is higher under Silent than Open-ended search. The silent client, therefore, is willing to continue the search into second round even when the transparent client will terminate it.

In addition to the cost of continuation, the provider's skill in problem-solving plays an important role in the client's decision. If the provider's skill in solving the problem is lower, the expected quality of the second solution is lower as well. There is a direct and an indirect reason for this. First, lower skill directly translates to a poorer distribution of results for a given level of effort. Lower skill also demotivates the provider from trying to find an acceptable solution. As a result, the client will find it valuable to extend the search into the second round only if the provider is sufficiently skilled.

Provider Behavior. The provider's efforts over the two rounds are also sensitive to the structure of the search process. The provider wants to find a solution to client's problem within the two rounds, and do this efficiently. The provider's quest for efficiency could lead to effort choices that are inefficient from the client's perspective. Recall that the provider's search cost increases in a convex fashion with his effort level and thus the expected quality of the generated solution. Therefore, it would be reasonable to expect that a provider would choose (almost) the same effort level in both rounds. However, such "smoothing" of efforts is efficient only if the client commissions two solutions in advance. As we saw in Proposition 8 above, the client's incentive to continue the search varies from one structure to another; this contributes to important differences between the provider's efforts in various structures. We derive valuable relationships between efforts in the two rounds in Proposition 9 below.

Proposition 9. *The provider's efforts depend on the search structure as follows:*

1. *In the second round, the provider exerts higher effort under the Committed and Open-ended (when $v_1 \geq A$) search processes than under the Silent search, i.e., $\mu_2^C = \mu_2^{O-} \geq \mu_2^S \geq \mu_2^{O+}$.*

2. *In the first round, the provider exerts higher effort under the Committed than Open-ended search, i.e., $\mu_1^C \geq \mu_1^O$, if and only if $-c\mu_0^2 + w \leq 0$.*

In the second round, unsurprisingly, the provider exerts the most effort when it is clear that the first solution was not acceptable. This occurs automatically in the Committed structure (when a second round is used only if the first solution fails) and in the Open-ended structure (where the client shares this bad news). The provider's second round effort in the Silent structure never exceeds his second round effort in the transparent structures (Open-ended or Committed). This is because the provider exerts less effort after accounting for the fact that his first solution *might* have exceeded the acceptability threshold A. We can expect that the client would stay away from ever using the Silent structure due to this relative lethargy of

the provider in the second round. There are, however, advantages to using the Silent search structure that yield over two rounds.

A more complete picture of the provider's efforts emerges when we consider the first round efforts. From Proposition 9(ii) above, we can see that the provider exerts greater first round effort in the Committed search than in the Open-ended search if – and only if – the per-round wage (w) is sufficiently low.[5] Interestingly, to understand this, recall that the client's internal threshold for stopping search in the Open-ended structure, v_s^O, is high when w is small (Proposition 3). This, in the first round, discourages the provider and leads to lower effort. Further insights about provider behavior can be derived from numerical comparisons.

Propositions 1 and 4 have already shown that provider is prone to procrastination, which delays the arrival of an acceptable solution relative to a scenario where the provider's efforts are smoothed over time. We define the *Procrastination Ratio* in a search structure as the amount of escalation in effort from the first round to the second; more specifically, it is the ratio of second round effort to first round effort of the provider.[6] In Figure 5.1, we compare the Procrastination Ratios of the three structures. It provides a valuable clue to understand the benefit the client receives from the Silent search structure. We observe that both the Committed and Open-ended structures (where the client provides feedback) lead to more procrastination by the provider; this is especially so when the problem is less difficult (low A). Why? When the client provides feedback, the provider can raise his second round effort if the first solution is unacceptable; this also allows the provider to intentionally reduce his first round effort. On the other hand, as we discussed in §4.3, this conditioned second round effort is not possible when the search is Silent. Further, exerting higher effort in the first round can force the client to be less demanding in Silent search. As a result, Silent search leads to less procrastination than either of the other structures.

This analysis offers insights regarding the behavior of the client and provider within various search structures. Now, we turn our attention to which structures are preferred by the client, and when.

5.2   Efficiency of Delegated Search

From the client's point of view, arguably the most important factor – outside of financial parameters – that determines the success of delegated search is the provider's ability, k. For the provider, the difficulty of the problem A, represents the desirability of a search engagement. In this section, we numerically compare the efficiency of the search process in terms of these two critical parameters.

Commit or Not Commit?   Recall that the client renounces all flexibility in the Committed search

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[5]Note that the provider would still have an incentive to participate if the fixed fee F is sufficiently high. See Lemma 7 in appendix.

[6]In the Committed and Open-ended cases, we use the second round effort exerted when the first round did not yield an acceptable result.
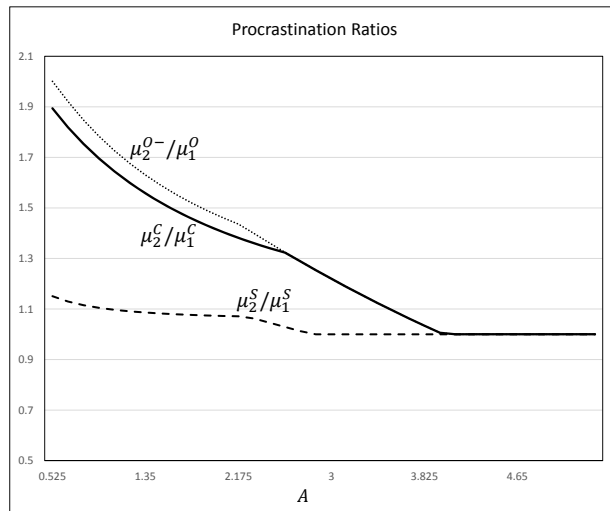
Figure 5.1: Procrastination under Different Search Structures
Parameters: The same as in Figure 4.1, except $\mu_0 = 0.45$, with $A \in [0,5]$.

by declaring upfront that an early breakthrough will terminate the search. While this seems inefficient in contexts where a provider too commits to an effort level upfront, we showed in §5.1 that the Client's commitment could have a positive influence on the the provider's behavior. In Figure 5.2, we compare the client's total payoff under the Open-ended and Committed search structures with respect to the provider's ability (k), and the difficulty of the problem (A). The "None" region represents the area (i.e., ranges of A and k) where it is optimal for the client to not participate in either of the search structures, and the "Both" region represents the area where the two search structures perform the same. As the figure illustrates, the Open-ended structure is not unequivocally better for the client. Indeed, it is dominant only if the problem that is being solved is relatively easy (low A) and the provider is not highly skilled (low k).

There are two reasons for the desirability of the Committed search when k or A is large. Suppose the client uses an Open-ended search structure. First, when the provider's ability is higher (large k), the client has a stronger belief that giving the provider another opportunity in the second round would result to a better solution. In other words, v_s^O increases with k, making the client appear harder to please under the Open-ended structure when k is high; this in turn discourages the provider more in the first round and exacerbates his procrastination. On the other hand, when the problem is more difficult to solve (large A), the provider's behavior reduces the value of the additional flexibility from Open-ended search. Recall that if the first solution is acceptable, the provider will exert only the minimal effort of μ_0 in the second round. The probability that outcome of such effort will exceed the acceptability threshold A naturally decreases with A. As a result, the client's marginal value of continuing the search decreases with A; this is reflected in the gap between threshold v_s^O and A (Proposition 3). At the same time, the Open-ended approach reduces

the provider's effort in the first round. This combination of factors results in the superiority of Committed search for large values of A or k.

It should be noted that for sufficiently difficult problems, it does not matter whether the client uses the Open-ended or Committed approach. Mathematically, v_s^O eventually trickles down to a level below A, meaning the client does not get any value out of the purported flexibility of Open-ended search. Intuitively, when A is high, the problem is so hard that there is only a small probability of getting an acceptable solution even when the search occurs over two rounds; and this probability shrinks further at the end of the first round. Therefore, the client sees no point in extending the search to a second round (at the cost of $c_I + w$) once an acceptable solution has been found.

Figure 5.2: Efficiency of Committed and Open-ended search processes
Parameters: The same as in Figure 4.1 with $A \in [0, 14]$ and $k \in [3, 8]$.

Provide Feedback or Remain Silent? The issue of providing feedback on the first solution assumes importance when the firm wants to retain flexibility to search again in the second round. On one hand, giving feedback is valuable in motivating the provider who underachieves in the first round, especially if he has high capability at solving the problem. On the other hand, remaining silent about the quality of the first solution can be useful in motivating the provider in the first round because he may see a greater first round effort as a positive way to force a conclusion to the search. Therefore, the Open-ended and Silent structures both have strengths, which appear at different temporal points of the search exercise. In Figure 5.3, we compare the client's total payoff under the Open-ended and Silent search structures with respect to

the provider's ability (k), and the difficulty of the problem (A). The "None" region represents the area (i.e., ranges of A and k) where it is optimal for the client to not participate in either of the search structures, and the "Both" region represents the area where the two search structures perform the same. As is illustrated in the figure, the providing feedback is not unequivocally better for the client. Indeed, it is dominant only if the problem that is being solved is relatively easy (low A) and the provider is highly skilled (high k).
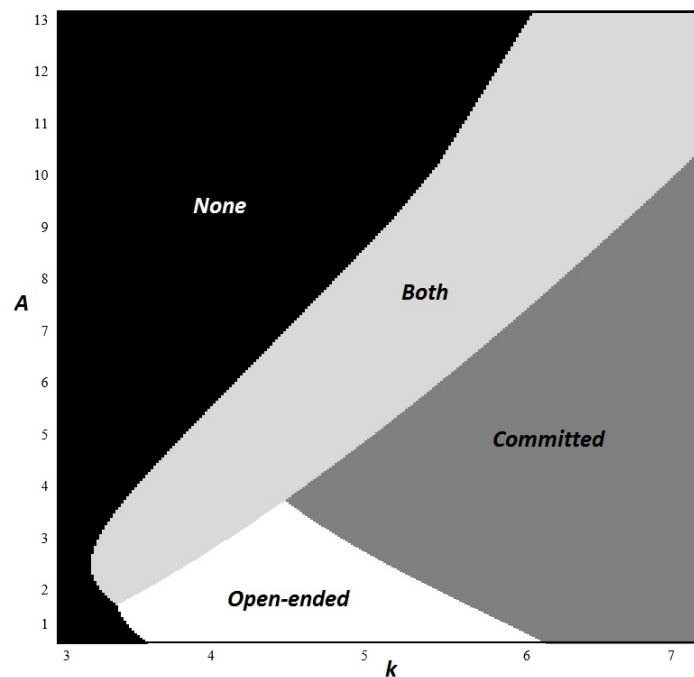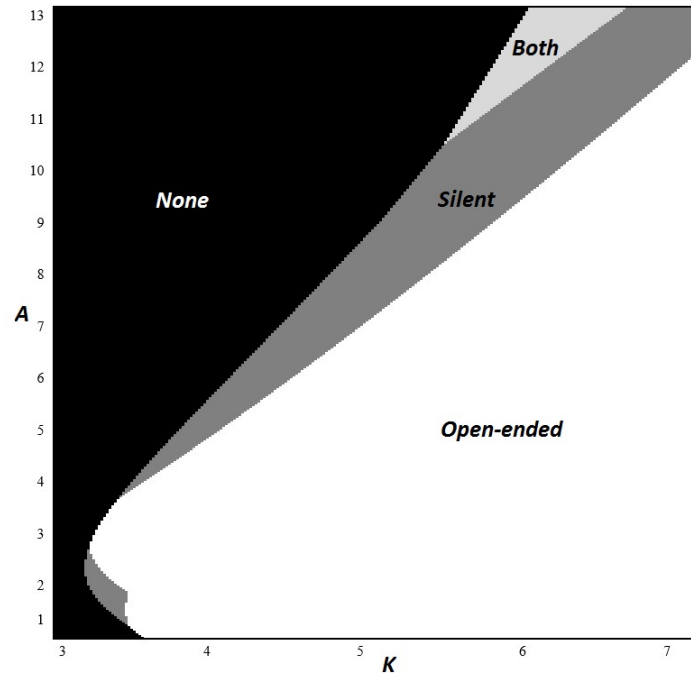

Figure 5.3: Efficiency of Silent and Open-ended search processes
Parameters: The same as in Figure 4.1 with $A \in [0, 13]$ and $k \in [3, 7]$.

If the provider is very skilled (high k) or if the problem is relatively easy to solve (small A, in general), then the client prefers to share information in the Open-ended approach. Otherwise, the client commits to remain Silent. To understand why such a preference emerges, consider a delegated search in which k is large and A is small. In this scenario, the provider – regardless of his effort in the first round – expects to find an acceptable solution in the first round. As a consequence of this belief, his effort in the second round would be minimal if he does not receive any feedback on the first round solution. This could be disastrous for the client if the first round solution does indeed turn out to be unacceptable. Therefore, in these situations where the problem is easy relative to the skill of the provider (low A, high k), the client uses the Open-ended approach as a protection against the complacency of the provider.

Why is the Silent search ever valuable? When the problem is very difficult to solve (high A), the client is more concerned about finding *at least one* acceptable solution than about the efficiency of finding such a solution. This escalates the client's cost from the provider's tendency to procrastinate in the Open-ended

structure. When the client commits to being Silent about the first solution, the provider is motivated to generate two meaningful solutions rather than two solutions of vastly different qualities. Therefore, when the problem is sufficiently hard, the client prefers the Silent structure.

6   Conclusions

Clients who face challenging problems in many realms delegate the search for solutions to skilled providers. Motivated by observations in several contexts such as marketing and product design, we studied a fundamental source of friction that affects these delegated search engagements. We develop a game-theoretic model where the delegated provider searches for a solution by generating a sequence of candidate solutions over a finite horizon (of two rounds). The solution generation process is stochastic, but the provider can improve the distribution of results in any round by exerting greater effort. The client evaluates the proposed solutions and decides whether or not to continue the search. We explore the equilibrium decisions of both the client (to terminate) and the provider (to exert effort), and analyze how they change over time.

Our model yields several interesting insights of managerial consequence. First, we uncover an interesting dynamic that exists in the relationship. Before the search begins, both parties in a delegated search context profess an enthusiasm to conclude the search as soon as possible because search is costly for both: the provider receives his fixed payment upon conclusion of the search, the client can implement the solution upon conclusion of the search. However, we show that both the client and the provider behave in a way that contributes to the extension of the project. The client might continue the search — even after an acceptable solution is proposed — because the provider may generate an even better solution. This behavior of the client could discourage the provider in the early round of search, which leads to a low effort from the provider. Ultimately, the client sees the provider as a procrastinator, the provider perceives the client as hard-to-please, and the search itself is inefficient (relative to a centralized version).

We explore the impact of the structure of delegated search on the outcome of the search process. We study the strategic behavior of clients and providers in delegated search processes and under three distinct structures: Committed, in which the client seeks a breakthrough; Open-ended, where the client reserves the right to search even after she tells the provider that a breakthrough has been achieved; and Silent, where the client does not provide any feedback until she is ready to implement a solution. These structures vary significantly in two important dimensions: the degree of flexibility the client has in continuing the search, and the level of transparency between the client and the provider. Comparing the efficiency of Committed and Open-ended search structures, we observe that the flexibility of the Open-ended structure dominates only if the problem that is being solved is relatively easy and the provider is not highly skilled. However,

comparing the efficiency of Open-ended and Silent search structures, we observe that it is beneficial for the client to be transparent and share information about previous solutions only when the provider is very skilled or when the problem is relatively easy to solve.

Future research can extend this work along several important dimensions. First, in this paper we focused on settings where the parameters of an acceptable solution can be communicated upfront. It will be useful to consider consumer-oriented settings, such as the search for a home or a partner, which will require a focus on subjective criteria for decision-making. This paper also focused on contexts where subsequent ideas are independently generated by the provider. In other contexts, which include the revision of industrial or academic research, the creative agent may build on a previously generated solution in subsequent iterations. We believe this provides an interesting opportunity for further research. Finally, we have observed that providers might choose to send dedicated teams for client engagements. This can be modeled as a provider's commitment to an effort level over the duration of search. Our preliminary analysis suggests that considering this behavioral commitment can lead to valuable insights for managers who delegate search.

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Appendix

1. Committed Search

We denote by $\mu_t^C \in [\mu_0, 1]$ the provider's equilibrium effort under the Committed search in round t for $t \in \{1, 2\}$. In addition, we denote by U_t^C and V_t^C, the provider's and client's payoff to-go, respectively, under the Committed search in round t for $t \in \{1, 2\}$.

Lemma 1. *Suppose both the client and provider participate in both rounds of the Committed search. Then, the provider's efforts are as follows:* $\mu_1^C = \max\left\{\mu_0, \frac{\left(F - U_2^C\right) \cdot e^{-A/k}}{2c}\right\}$ *and* $\mu_2^C = \max\left\{\mu_0, \frac{F \cdot e^{-A/k}}{2c}\right\}$.

Proof. Using backward induction, we first characterize the provider's effort in the second round, and then use that result to characterize the provider's effort in the first round. In the second round, the provider chooses his effort by maximizing his expected payoff as follows:

$$
\begin{aligned}
\mu_2^C &= \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p_\mu(v_2 \geq A) \cdot F \\
&= \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + \mu \cdot e^{-A/k} \cdot F,
\end{aligned}
$$

which is concave in μ. Thus, by the first order condition and the assumption that $\mu_2^C \geq \mu_0$, we obtain $\mu_2^C = \max\left\{\mu_0, \frac{F \cdot e^{-A/k}}{2c}\right\} > 0$. Accordingly, the provider's payoff in the second round is $U_2^C = -c \cdot \left(\mu_2^C\right)^2 + w + \mu_2^C \cdot e^{-A/k} F$.

Similarly, in the first round, the provider chooses his effort by maximizing her expected payoff to-go, while anticipating his payoff in the second round, as follows:

$$
\begin{aligned}
\mu_1^C &= \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p_\mu(v_1 \geq A) \cdot F + (1 - p_\mu(v_1 \geq A)) \cdot U_2^C \\
&= \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + \mu \cdot e^{-A/k} \cdot F + \left(1 - \mu \cdot e^{-A/k}\right) \cdot U_2^C,
\end{aligned}
$$

which is concave in μ. Thus, by the first order condition and the assumption that $\mu_1^C \geq \mu_0$, we obtain $\mu_1^C = \max\left\{\mu_0, \frac{\left(F - U_2^C\right) \cdot e^{-A/k}}{2c}\right\}$. Accordingly, the provider's payoff in the first round is $U_1^C = -c \cdot \left(\mu_1^C\right)^2 + w + \mu_1^C \cdot e^{-A/k} \cdot F + \left(1 - \mu_1^C \cdot e^{-A/k}\right) \cdot U_2^C$. $\square$

Lemma 2. *[Provider's participation] Under the Committed search, there exists a threshold* $F_{lb} \doteq \frac{c \cdot \mu_0^2 - w}{\mu_0 \cdot e^{-A/k}}$ *such that for* $F \geq F_{lb}$, $U_1^C \geq U_2^C \geq 0$.

Proof. By the optimality condition, we obtain $U_2^C = -c \cdot \left(\mu_2^C\right)^2 + w + \mu_2^C \cdot e^{-A/k} F \geq -c \cdot (\mu_0)^2 + w + \mu_0 \cdot e^{-A/k} F$. Hence, $U_2^C \geq 0$ if $-c \cdot (\mu_0)^2 + w + \mu_0 \cdot e^{-A/k} F \geq 0$, and the latter holds if and only if $F \geq F_{lb}$.

By the optimality condition, we obtain $U_1^C = -c \cdot \left(\mu_1^C\right)^2 + w + \mu_1^C \cdot e^{-A/k} \cdot F + \left(1 - \mu_1^C \cdot e^{-A/k}\right) \cdot U_2^C \geq$ $-c \cdot \left(\mu_2^C\right)^2 + w + \mu_2^C \cdot e^{-A/k} \cdot F + \left(1 - \mu_2^C \cdot e^{-A/k}\right) \cdot U_2^C = U_2^C + \left(1 - \mu_2^C \cdot e^{-A/k}\right) \cdot U_2^C \geq U_2^C$, in which the last inequality holds because $\mu_2^C \cdot e^{-A/k} \leq 1$ and $U_2^C \geq 0$ for $F \geq F_{lb}$. $\qquad\square$

Lemma 3. *[Client's participation] Under the Committed search, there exists a threshold $F_{hb} \doteq A + k - \frac{c_I + w}{\mu_0 \cdot e^{-A/k}}$ such that, if $v_1 < A$, the client continues the search to the second round (i.e., $\mathbb{E}_{\mu_2^C}\left[V_2^C\right] \geq 0$) for $F \leq F_{hb}$.*

Proof. We obtain $\mathbb{E}_{\mu_2^C}\left[V_2^C\right] = -c_I - w + p_{\mu_2^C}(v_2 \geq A) \cdot \left(\mathbb{E}_{\mu_2^C}\left[v_2 \mid v_2 \geq A\right] - F\right) = -c_I - w + \mu_2^C \cdot e^{-A/k} \cdot$ $\left(\mathbb{E}_{\mu_2^C}\left[v_2 \mid v_2 \geq A\right] - F\right)$. Moreover, because $p(v_t \leq A) = 1 - \mu_t^C \cdot e^{-A/k}$, we obtain

$$\mathbb{E}_{\mu_t^C}\left[v_t \mid v_t \geq A\right] = \frac{\int_A^{+\infty} \left(\frac{\mu_t^C}{k}\right) \cdot e^{-v/k} \cdot v \cdot dv}{\mu_t^C \cdot e^{-A/k}} = A + k, \tag{6.1}$$

which is consistent with the memory-less property of the exponential distribution. Thus, because $\mu_2^C \geq \mu_0$, we obtain $\mathbb{E}_{\mu_2^C}\left[V_2^C\right] = -c_I - w + \mu_2^C \cdot e^{-A/k} \cdot (A + k - F) \geq -c_I - w + \mu_0 \cdot e^{-A/k} \cdot (A + k - F)$. Hence, the client continues the search when $-c_I - w + \mu_0 \cdot e^{-A/k} \cdot (A + k - F) \geq 0$, which holds if and only if $F \leq F_{hb}$. $\qquad\square$

Proof of Proposition 1: By Lemmas 2 and 3, for any $F \in [F_{lb}, F_{hb}]$, both the client and provider participate in both rounds of the Committed search. In addition, by Lemma 1, $\mu_2^C = \max\left\{\mu_0, \frac{F \cdot e^{-A/k}}{2c}\right\} \geq \max\left\{\mu_0, \frac{\left(F - U_2^C\right) \cdot e^{-A/k}}{2c}\right\} = \mu_1^C$, given that $U_2^C \geq 0$ by Lemma 2.$\square$

Proof of Proposition 2: (i) By Proposition 1, $\mu_2^C = \max\left\{\mu_0, \frac{F \cdot e^{-A/k}}{2c}\right\}$. Let us define $f_2^C(A) \doteq \frac{F \cdot e^{-A/k}}{2c}$. We obtain $\frac{df_2^C(A)}{dA} = \frac{-F \cdot e^{-A/k}}{2kc} < 0$. Because μ_0 is independent of A and $f_2^C(A)$ is decreasing in A, we conclude that μ_2^C is decreasing in A. In addition, by solving $f_2^C(A) = \mu_0$, we obtain that there exists a unique threshold $A_2^C \doteq k \cdot \ln\left(\frac{F}{2\mu_0 c}\right)$ such that $\mu_2^C = \mu_0$ if and only if $A \geq A_2^C$, and $\mu_2^C = f_2^C(A) > \mu_0$ otherwise.

(ii) By Proposition 1, $\mu_1^C = \max\left\{\mu_0, \frac{\left(F - U_2^C\right) \cdot e^{-A/k}}{2c}\right\} \geq \mu_0$. Because by Proposition 1, $\mu_1^C \leq \mu_2^C$, for $A \geq A_2^C$, $\mu_0 \leq \mu_1^C \leq \mu_2^C = \mu_0$. This implies that $\mu_1^C = \mu_0$ for $A \geq A_2^C$. We next consider $A < A_2^C$. By part (i), $\mu_2^C = \frac{F \cdot e^{-A/k}}{2c}$. Accordingly, $U_2^C = -c \cdot \left(\mu_2^C\right)^2 + w + \mu_2^C \cdot e^{-A/k} F = w + \frac{F^2 \cdot e^{-2A/k}}{4c}$. Replacing that in μ_1^C, we obtain $\mu_1^C = \max\left\{\mu_0, \frac{\left(F - w - \frac{F^2 \cdot e^{-2A/k}}{4c}\right) \cdot e^{-A/k}}{2c}\right\}$. Let us define $f_1^C(A) \doteq \frac{\left(F - w - \frac{F^2 \cdot e^{-2A/k}}{4c}\right) \cdot e^{-A/k}}{2c}$. We obtain:

$$\frac{df_1^C(A)}{dA} = \frac{e^{-A/k}\left(3F^2 \cdot e^{-2A/k} - 4Fc + 4wc\right)}{8kc^2}.$$

By solving $\frac{df_1^C(A)}{dA} = 0$, we obtain that that there exists a unique threshold $A^C \doteq \frac{k}{2} \cdot \ln\left(\frac{3F^2}{4c(F-w)}\right)$ such that $\frac{df_1^C(A)}{dA} \geq 0$ if and only if $A \leq A^C$. This implies that $f_1^C(A)$ is unimodal in A. In addition, $f_1^C(0) =$

$\frac{3F^2-4c(F-w)}{8ck^2}$. We therefore consider two possible cases:

(a) Suppose $f_1^C(0) < 0$, i.e., $\left(3F^2 - 4c\,(F - w)\right) < 0$. Then, $f_1^C(A) = \mu_0$ has exactly two positive solutions. We define $\left(A_{1l}^C, A_{1h}^C\right)$ as the two positive solutions. We therefore obtain $\mu_1^C = \mu_0$ for $A \leq A_{1l}^C$ and $A \geq A_{1h}^C$, and $\mu_1^C = f_1^C(A) > \mu_0$ for $A_{1l}^C < A < A_{1h}^C$.

(b) Suppose $f_1^C(0) \geq 0$, i.e., $\left(3F^2 - 4c\,(F - w)\right) \geq 0$. Then, $f_1^C(A) = \mu_0$ has only one positive solution. We define A_{1h}^C as its only positive solution. We therefore obtain $\mu_1^C = \mu_0$ for $A \geq A_{1h}^C$, and $\mu_1^C = f_1^C(A) > \mu_0$ for $A < A_{1h}^C$. $\square$

2. Open-ended Search

We denote by $\left(\mu_1^O, \mu_2^{O-}, \mu_2^{O+}\right)$ the provider's equilibrium effort under the Open-ended search in the first round, in the second round when $v_1 < A$, and in the second round when $v_1 \geq A$, respectively. In addition, we denote by U_t^O and V_t^O, the provider's and client's payoff to-go, respectively, under the Open-ended search in round t for $t \in \{1, 2\}$.

Lemma 4. *Under the Open-ended search, suppose the client chooses to continue the search to the second round and the provider participates. Then, (i) if $v_1 \geq A$, $\mu_2^{O+} = \mu_0$, and (ii) if $v_1 < A$, $\mu_2^{O-} = \max\left\{\mu_0, \frac{F \cdot e^{-A/k}}{2c}\right\}$.*

Proof. (i) In the second round and when $v_1 \geq A$, the provider's payment F is guaranteed in the second round irrespective of the quality of the second outcome. He therefore chooses his effort as follows: $\mu_2^{O+} = \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + F$. Since the provider's payoff is decreasing in μ and by assumption $\mu_2^{O+} \geq \mu_0$, we obtain $\mu_2^{O+} = \mu_0$. Accordingly, the provider's payoff in the second round is $U_2^{O+} = -c \cdot (\mu_0)^2 + w + F$.

(ii) In the second round and when $v_1 < A$, the provider chooses his effort as follows:

$$
\begin{aligned}
\mu_2^{O-} &= \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p_\mu(v_2 \geq A) \cdot F \\
&= \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + \mu \cdot e^{-A/k} \cdot F,
\end{aligned}
$$

which is concave in μ. Thus, by the first order condition and the assumption that $\mu_2^{O-} \geq \mu_0$, we obtain $\mu_2^{O-} = \max\left\{\mu_0, \frac{F \cdot e^{-A/k}}{2c}\right\} > 0$. Accordingly, the provider's payoff in the second round is $U_2^{O-} = -c \cdot \left(\mu_2^{O-}\right)^2 + w + \mu_2^{O-} \cdot e^{-A/k} F$. $\square$

Lemma 5. *Consider the Open-ended search and suppose that $v_1 \geq A$. Then, there exists a threshold $v_s^O \doteq k \cdot \ln\left(\frac{\mu_0 k}{c_I + w}\right)$ such that it is optimal for the client to stop the search if and only if $v_1 \geq \max\left\{A, v_s^O\right\}$.*

Proof. When $v_1 \geq A$, the client chooses to continue or stop the search by solving the following maximization

problem:

$$
\begin{aligned}
V_2^{O+} &= \max\left\{(v_1 - F), \mathbb{E}_{\mu_2^{O+}}\left[V_2^{O+}\left(\mu_2^{O+}\right)\right]\right\} \\
&= \max\left\{(v_1 - F), -c_I - w + p_{\mu_2^{O+}}(v_2 \geq v_1) \cdot \left(\mathbb{E}_{\mu_2^{O+}}[v_2 \mid v_2 \geq v_1] - F\right) + \left(1 - p_{\mu_2^{O+}}(v_2 \geq v_1)\right) \cdot (v_1 - F)\right\}.
\end{aligned}
$$

By Lemma 4, $\mu_2^{O+} = \mu_0$. Thus, similar to (6.1), $\mathbb{E}_{\mu_2^{O+}}[v_2 \mid v_2 \geq v_1] = v_1 + k$. We therefore obtain,

$$
\begin{aligned}
V_2^{O+} &= \max\left\{(v_1 - F), -c_I - w + p_{\mu_2^{O+}}(v_2 \geq v_1) \cdot \left(\mathbb{E}_{\mu_2^{O+}}[v_2 \mid v_2 \geq v_1] - F\right) + \left(1 - p_{\mu_2^{O+}}(v_2 \geq v_1)\right) \cdot (v_1 - F)\right\} \\
&= \max\left\{(v_1 - F), -c_I - w + \mu_0 \cdot k \cdot e^{-v_1/k} + v_1 - F\right\},
\end{aligned}
$$

which implies the client stops the search if and only if $-c_I - w + \mu_0 \cdot k \cdot e^{-v_1/k} \leq 0$, which is decreasing in v_1. Hence, there exists a unique threshold $v_s^O \doteq k \cdot \ln\left(\frac{\mu_0 k}{c_I + w}\right)$ such that $-c_I - w + \mu_0 \cdot k \cdot e^{-v_1/k} \leq 0$ if and only if $v_1 \geq v_s^O$. In addition, because by assumption $v_1 \geq A$, the client stops the search if $v_1 \geq \max\left\{A, v_s^O\right\}$ and continue the search if $A \leq v_1 < \max\left\{A, v_s^O\right\}$. $\qquad\square$

Lemma 6. *Suppose both the client and provider participate in both rounds of the Open-ended search. Then, the provider's efforts are as follows:* $\mu_1^O = \max\left\{\mu_0, \dfrac{\left(e^{-A/k} - e^{-\max\left\{A, v_s^O\right\}/k}\right) \cdot \left(-c\mu_0^2 + w\right) + e^{-A/k} \cdot \left(F - U_2^{O-}\right)}{2c}\right\},$
$\mu_2^{O+} = \mu_0$, *and* $\mu_2^{O-} = \max\left\{\mu_0, \dfrac{F \cdot e^{-A/k}}{2c}\right\}$, *where* $U_2^{O-} = -c \cdot \left(\mu_2^{O-}\right)^2 + w + \mu_2^{O-} \cdot e^{-A/k}F$.

Proof. The provider's efforts in the second round (i.e., μ_2^{O+} and μ_2^{O-}) are characterized in Lemma 4. In addition, by Lemma 5, the client stops the search and implements the solution of the first round if and only if $v_1 \geq \max\left\{A, v_s^O\right\}$. Anticipating these, the provider chooses her effort in the first round by solving the following:

$$
\begin{aligned}
\mu_1^O &= \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p_\mu(v_1 \geq \max\left\{A, v_s^O\right\}) \cdot F \\
&\quad + p_\mu(A \leq v_1 < \max\left\{A, v_s^O\right\}) \cdot U_2^{O+} + p_\mu(v_1 < A) \cdot U_2^{O-},
\end{aligned}
$$

replacing U_2^+ and U_2^- from Lemma 4, we obtain

$$
\begin{aligned}
\mu_1^O \quad = \quad & \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p_\mu(v_1 \geq \max\{A, v_s^O\}) \cdot F \\
& + p_\mu(A \leq v_1 < \max\{A, v_s^O\}) \cdot \left(-c\mu_0^2 + w + F\right) + p_\mu(v_1 < A) \cdot U_2^{O-} \\
= \quad & \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + \mu \cdot e^{-\max\{A, v_s^O\}/k} \cdot F \\
& + \mu \cdot \left(e^{-A/k} - e^{-\max\{A, v_s^O\}/k}\right) \cdot \left(-c\mu_0^2 + w + F\right) + \left(1 - \mu e^{-A/k}\right) \cdot U_2^{O-} \\
= \quad & \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + U_2^{O-} \\
& + \mu \cdot \left[\left(e^{-A/k} - e^{-\max\{A, v_s^O\}/k}\right) \cdot \left(-c\mu_0^2 + w\right) + e^{-A/k} \cdot \left(F - U_2^{O-}\right)\right], \qquad (6.2)
\end{aligned}
$$

which is concave in μ. Thus, by the first order conditions and the assumption that $\mu_1^O \geq \mu_0$, we obtain

$$
\mu_1^O = \max\left\{\mu_0, \frac{\left(e^{-A/k} - e^{-\max\{A, v_s^O\}/k}\right) \cdot \left(-c\mu_0^2 + w\right) + e^{-A/k} \cdot \left(F - U_2^{O-}\right)}{2c}\right\}. \qquad \square
$$

Lemma 7. *[Provider's participation] Under the Open-ended search, there exist thresholds F_{lb} (as defined in Lemma 2) and $F_{lb}^O \doteq \frac{2c\mu_0}{e^{-A/k}} > F_{lb}$ such that (i) $U_2^{O-} \geq 0$ and $U_2^{O+} \geq 0$ for $F \geq F_{lb}$. (ii) If $-c\mu_0^2 + w \geq 0$, $U_1^O \geq 0$ for $F \geq F_{lb}$, and if $-c\mu_0^2 + w < 0$, then $U_1^O \geq 0$ for $F > F_{lb}^O$.*

Proof. (i) By Proposition 1 and Lemma 6, $\mu_2^{O-} = \mu_2^C$. Thus, $U_2^{O-} = -c \cdot \left(\mu_2^{O-}\right)^2 + w + \mu_2^{O-} \cdot e^{-A/k} F = U_2^C$. In addition, by Lemma 2, $U_2^{O-} = U_2^C \geq 0$ for all $F \geq F_{lb}$. We next show the result for U_2^{O+}. By Lemma 6, $\mu_2^{O+} = \mu_0$ and $U_2^{O+} = -c\mu_0^2 + w + F$. Then, if $-c\mu_0^2 + w \geq 0$, $U_2^{O+} \geq 0$, and if $-c\mu_0^2 + w < 0$, $U_2^{O+} \geq 0$ for any $F \geq F_{lb} = \frac{c \cdot \mu_0^2 - w}{\mu_0 \cdot e^{-A/k}} \geq c\mu_0^2 - w$, in which the last inequality holds because $\mu_0 \cdot e^{-A/k} \leq 1$ and $c\mu_0^2 - w > 0$.

(ii) Suppose $-c\mu_0^2 + w \geq 0$. By (6.2), we obtain

$$
\begin{aligned}
U_1^O \quad = \quad & \max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p_\mu(v_1 \geq \max\{A, v_s^O\}) \cdot F \\
& + p_\mu(A \leq v_1 < \max\{A, v_s^O\}) \cdot \left(-c\mu_0^2 + w + F\right) + p_\mu(v_1 < A) \cdot U_2^{O-} \\
\geq \quad & \max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p_\mu(v_1 \geq A) \cdot F + p_\mu(v_1 < A) \cdot U_2^{O-} \\
\geq \quad & -c\left(\mu_2^{O-}\right)^2 + w + p_{\mu_2^{O-}}(v_1 \geq A) \cdot F + p_{\mu_2^{O-}}(v_1 < A) \cdot U_2^{O-} \\
= \quad & U_2^{O-} + p_{\mu_2^{O-}}(v_1 < A) \cdot U_2^{O-} \geq U_2^{O-},
\end{aligned}
$$

in which the first inequality holds because by assumption $-c\mu_0^2 + w \geq 0$ (i.e., $-c\mu_0^2 + w + F \geq F$), and the second inequality holds by the optimality condition. Hence, by part (i), $U_1^O \geq U_2^{O-} \geq 0$ for all $F > F_{lb}$.

Suppose $-c\mu_0^2 + w < 0$. By (6.2), we obtain

$$
\begin{aligned}
U_1^O &= \max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p_\mu(v_1 \geq \max\{A, v_s^O\}) \cdot F \\
&\quad + p_\mu(A \leq v_1 < \max\{A, v_s^O\}) \cdot \left(-c\mu_0^2 + w + F\right) + p_\mu(v_1 < A) \cdot U_2^{O-} \\
&\geq -c\left(\mu_2^{O-}\right)^2 + w + p_{\mu_2^{O-}}(v_1 \geq \max\{A, v_s^O\}) \cdot F \\
&\quad + p_{\mu_2^{O-}}(A \leq v_1 < \max\{A, v_s^O\}) \cdot \left(-c\mu_0^2 + w + F\right) + p_{\mu_2^{O-}}(v_1 < A) \cdot U_2^{O-} \\
&> -c \cdot \left(\mu_2^{O-}\right)^2 + w + p_{\mu_2^{O-}}(v_1 \geq A) \cdot \left(-c\mu_0^2 + w + F\right) + p_{\mu_2^{O-}}(v_1 < A) \cdot U_2^{O-}, \quad (6.3)
\end{aligned}
$$

in which the first inequality holds by the optimality condition. The second inequality holds because by assumption $-c\mu_0^2 + w < 0$ (i.e., $F > -c\mu_0^2 + w + F$). It can be seem that for $F \geq F_{lb}^O$, $\mu_2^{O-} = \frac{Fe^{-A/k}}{2c} \geq \mu_0$. Thus, by replacing μ_2^{O-} in (6.3), we obtain

$$
\begin{aligned}
U_1^O &\geq -c \cdot \left(\mu_2^{O-}\right)^2 + w + p_{\mu_2^{O-}}(v_1 \geq A) \cdot \left(-c\mu_0^2 + w + F\right) + p_{\mu_2^{O-}}(v_1 < A) \cdot U_2^{O-} \\
&= -c \cdot \left(\mu_2^{O-}\right)^2 + w + p_{\mu_2^{O-}}(v_1 \geq A)F + p_{\mu_2^{O-}}(v_1 \geq A) \cdot \left(-c\mu_0^2 + w\right) + p_{\mu_2^{O-}}(v_1 < A) \cdot U_2^{O-} \\
&= w + c \cdot \left(\mu_2^{O-}\right)^2 + p_{\mu_2^{O-}}(v_1 \geq A) \cdot \left(-c\mu_0^2 + w\right) + p_{\mu_2^{O-}}(v_1 < A) \cdot U_2^{O-} \\
&\geq w \cdot \left(1 + p_{\mu_2^{O-}}(v_1 \geq A)\right) + \left(1 - p_{\mu_2^{O-}}(v_1 \geq A)\right) c \cdot (\mu_0)^2 + p_{\mu_2^{O-}}(v_1 < A) \cdot U_2^{O-} \geq 0,
\end{aligned}
$$

in which , the second inequality holds because $\mu_2^{O-} \geq \mu_0$ for $F \geq F_{lb}^O$, and the last inequality holds because $1 - p_{\mu_1^O}(v_1 \geq A) \geq 0$ and $U_2^{O-} \geq 0$ for all $F \geq F_{lb}^O > F_{lb}$. $\qquad \square$

Lemma 8. *[Client's participation] Under Open-ended search, there exists a threshold F_{hb} (as defined in Lemma 3) such that, if $v_1 < A$, the client continues the search to the second round (i.e., $\mathbb{E}_{\mu_2^{O-}}\left[V_2^{O-}\right] \geq 0$) for $F \leq F_{hb}$.*

Proof. By Proposition **1** and Lemma 6, $\mu_2^{O-} = \mu_2^C$. Thus, the proof follows the same as in Lemma 3. Note that we have already characterized the client's participation when $v_1 \geq A$ in Lemma 5. $\qquad \square$

Proof of Proposition 3: By Lemma 5, it is optimal for the client to stop the search after the first round if $v_1 \geq \max\{A, v_s^O\}$ and continue the search if $A \leq v_1 < \max\{A, v_s^O\}$. In addition, by Lemma 8, for any $v_1 < A$, the client continues the search to the second round for all $F \leq F_{hb}$. Hence, the client continues the search if $v_1 < \max\{A, v_s^O\}$ and stop the search if $v_1 \geq \max\{A, v_s^O\}$. $\square$

Proof of Proposition 4: By Lemmas 7 and 8, for any $F \in \left[F_{lb}^O, F_{hb}\right]$, both the client and provider participate in both rounds of the Open-ended search. In addition, by Lemma 6, $\mu_2^{O+} = \mu_0$ and $\mu_2^{O-} =$

$\max\left\{\mu_0, \frac{F\cdot e^{-A/k}}{2c}\right\} \geq \mu_2^{O+}$, and $\mu_1^O = \max\left\{\mu_0, \frac{\left(e^{-A/k}-e^{-\max\left\{A,v_s^O\right\}/k}\right)\cdot\left(-c\mu_0^2+w\right)+e^{-A/k}\cdot\left(F-U_2^{O-}\right)}{2c}\right\} \geq \mu_0 =$

μ_2^{O+}. We next compare μ_1^O and μ_2^{O-}. We consider three possible cases:

(a) Suppose $v_s^O \leq A$. Then, $\left(e^{-A/k} - e^{-\max\left\{A,v_s^O\right\}/k}\right) = 0$ and as a result, $\mu_1^O = \max\left\{\mu_0, \frac{e^{-A/k}\cdot\left(F-U_2^{O-}\right)}{2c}\right\} \leq$

$\max\left\{\mu_0, \frac{F\cdot e^{-A/k}}{2c}\right\} = \mu_2^{O-}$, because by Lemma 7, $U_2^{O-} \geq 0$ for all $F > F_{lb}$ and $F_{lb}^O > F_{lb}$.

(b) Suppose $v_s^O > A$ and $-c\mu_0^2 + w \leq 0$. Then, $\left(e^{-A/k} - e^{-,v_s^O/k}\right) > 0$ and because $-c\mu_0^2 + w \leq 0$, $\mu_1^O =$

$\max\left\{\mu_0, \frac{\left(e^{-A/k}-e^{-\max\left\{A,v_s^O\right\}/k}\right)\cdot\left(-c\mu_0^2+w\right)+e^{-A/k}\cdot\left(F-U_2^{O-}\right)}{2c}\right\} \leq \max\left\{\mu_0, \frac{e^{-A/k}\cdot\left(F-U_2^{O-}\right)}{2c}\right\} \leq \max\left\{\mu_0, \frac{F\cdot e^{-A/k}}{2c}\right\} =$

μ_2^{O-}, in which the last inequality holds because by Lemma 7, $U_2^{O-} \geq 0$ for all $F \geq F_{lb}$ and $F_{lb}^O > F_{lb}$.

(c) Suppose $v_s^O > A$ and $-c\mu_0^2 + w > 0$. Then for all $F > F_{lb}^O$, $\mu_2^{O-} = \frac{F\cdot e^{-A/k}}{2c} > \mu_0$. Thus, $U_2^{O-} = w + c\cdot\left(\mu_2^{O-}\right)^2$. Replacing U_2^{O-} in μ_1^O, we obtain

$$
\begin{aligned}
\mu_1^O &= \max\left\{\mu_0, \frac{\left(e^{-A/k} - e^{-\max\left\{A,v_s^O\right\}/k}\right)\cdot\left(-c\mu_0^2+w\right)+e^{-A/k}\cdot\left(F-U_2^{O-}\right)}{2c}\right\} \\
&= \max\left\{\mu_0, \frac{\left(e^{-A/k} - e^{-\max\left\{A,v_s^O\right\}/k}\right)\cdot\left(-c\mu_0^2+w\right)+e^{-A/k}\cdot\left(F-w-c\left(\mu_2^{O-}\right)^2\right)}{2c}\right\} \\
&= \max\left\{\mu_0, \frac{\left(-e^{-\max\left\{A,v_s^O\right\}/k}\right)\cdot\left(-c\mu_0^2+w\right)+e^{-A/k}\cdot\left(-c\left(\mu_2^{O-}\right)^2-c\mu_0^2\right)+F\cdot e^{-A/k}}{2c}\right\} \leq \mu_2^{O-},
\end{aligned}
$$

because by assumption $-c\mu_0^2 + w > 0$.$\square$

Proof of Proposition 5: (i)-(ii) By Proposition 4, $\mu_2^{O+} = \mu_0$, which is independent of A. In addition, $\mu_2^{O-} = \mu_2^C = \max\left\{\mu_0, \frac{F\cdot e^{-A/k}}{2c}\right\}$. Thus, similar to Proposition 2, μ_2^{O-} is decreasing in A, and $\mu_2^{O-} = \mu_0$ if and only if $A \geq A_2^C \doteq k\cdot\ln\left(\frac{F}{2\mu_0 c}\right)$. (iii) By Proposition 4, $\mu_1^O \leq \mu_2^{O-}$. Hence, for $A \geq A_2^C$, , which implies that $\mu_1^O = \mu_0$ for $A \geq A_2^C$. We next consider $A < A_2^C$. By part (i), $\mu_2^{O-} = \frac{F\cdot e^{-A/k}}{2c}$, which implies $U_2^{O-} = w + c\cdot\left(\mu_2^{O-}\right)^2 = w + \frac{F^2\cdot e^{-2A/k}}{4c}$. There are two possible cases:

(a) Suppose $v_s^O \leq A$. Then, $\mu_1^O = \mu_1^C$ and the proof follows the same as in Proposition 2.

(b) Suppose $v_s^O > A$. Then, by Proposition 5, $e^{-\max\left\{A,v_s^O\right\}/k} = \frac{c_I+w}{\mu_0 k}$. Replacing U_2^{O-} and v_s^O, we obtain

$$
\begin{aligned}
\mu_1^O &= \max\left\{\mu_0, \frac{\left(e^{-A/k} - e^{-\max\left\{A,v_s^O\right\}/k}\right)\cdot\left(-c\mu_0^2+w\right)+e^{-A/k}\cdot\left(F-U_2^{O-}\right)}{2c}\right\} \\
&= \max\left\{\mu_0, \frac{\left(\frac{-F^2}{4c}\right)\cdot\left(e^{-A/k}\right)^3 + \left(F-c\mu_0^2\right)\cdot\left(e^{-A/k}\right) + \left(\frac{c_I+w}{\mu_0 k}\right)\cdot\left(c\mu_0^2-w\right)}{2c}\right\}. \quad (6.4)
\end{aligned}
$$

Using (6.4), we define $f_1^O(A) \doteq \dfrac{\left(\frac{-F^2}{4c}\right)\cdot\left(e^{-A/k}\right)^3+\left(F-c\mu_0^2\right)\cdot\left(e^{-A/k}\right)+\left(\frac{c_I+w}{\mu_0 k}\right)\cdot\left(c\mu_0^2-w\right)}{2c}$. We then obtain:

$$\frac{df_1^O(A)}{dA} = \frac{-e^{-A/k}\left(3F^2 \cdot e^{-2A/k} - 4c^2\mu_0^2 + 4Fc\right)}{8kc^2}.$$

By solving $\frac{df_1^O(A)}{dA} = 0$, we obtain that that there exists a unique threshold $A^O \doteq \frac{k}{2} \cdot \ln\left(\frac{3F^2}{4c\left(c\mu_0^2-F\right)}\right)$ such that $\frac{df_1^O(A)}{dA} \geq 0$ if and only if $A \leq A^O$. This implies that $f_1^O(A)$ is unimodal in A. In addition, $f_1^O(0) = \frac{-4c^2 k\mu_0^3 + Fk(F+4c)\mu_0 + 4(c_I+w)(c\mu_0^2-w)c}{ck\mu_0}$. We therefore consider two possible cases:

(b-1) Suppose $f_1^O(0) < 0$. Then, $f_1^O(A) = \mu_0$ has exactly two positive solutions. We define $\left(A_{1l}^O, A_{1h}^O\right)$ as the two positive solutions to $f_1^O(A) = \mu_0$. We therefore obtain $\mu_1^O = \mu_0$ for $A \leq A_{1l}^O$ and $A \geq A_{1h}^O$, and $\mu_1^O = f_1^O(A) > \mu_0$ for $A_{1l}^O < A < A_{1h}^O$.

(b-2) Suppose $f_1^O(0) \geq 0$. Then, $f_1^O(A) = \mu_0$ has only one positive solution. We define A_{1h}^O as the only positive solution to $f_1^O(A) = \mu_0$. We therefore obtain $\mu_1^O = \mu_0$ for $A \geq A_{1h}^O$, and $\mu_1^O = f_1^O(A) > \mu_0$ for $A < A_{1h}^O$. $\square$

Lemma 9. *Let $\hat{\alpha} = 1 - \frac{c\mu_0^2-w}{F}$. For all values of $\alpha \in [0,\hat{\alpha}]$, the provider and client make the same decisions as the Open-ended search process. For all values of $\alpha \in [\hat{\alpha},1]$, the provider and client make the same decisions as the Committed search process.*

Proof. As shown in Proposition 3, the behavioral differences between the contracts arise due when $A \leq v_1 \leq v_s^O$; this is because of the client's decision to search further in some contracts and terminate in others. Therefore, we will focus on the event that $v_1 \geq A$ to prove the equivalence within subsets of contracts. First, regardless of the type of search and contract parameters, the provider will not exceed more than effort μ_0 in this case.

Suppose $\alpha \in [0,\hat{\alpha}]$. Similar to Proposition 3, it is straightforward to show that if $v_1 \geq A$: (i) $v_s^O = k\ln\left(\frac{\mu_0 k}{c_I+w}\right)$, which is independent of α; (ii) $c\mu_0^2 \leq (1-\alpha)F+w$, implying that the provider has an incentive to participate in the search process if the search continues into the second round. Therefore, in this case, the payoffs and behaviors of the parties are identical to the pure Open-ended search with $\bar{\alpha} = 0$ and $\bar{F} = \frac{F}{1-\alpha}$.

Suppose $\alpha \in [\hat{\alpha},1]$. Here $c\mu_0^2 > (1-\alpha)F+w$, meaning the provider will not participate in a second round of search if $v_1 \geq A$. Therefore, even if the provider seeks a potentially better solution, they will not receive one in the second round. This is, therefore, equivalent to the pure committed search with $\bar{\alpha} = 1$ and $\bar{F} = \frac{F}{1-\alpha}$. Therefore, it is sufficient to consider only pure Committed and Open-ended search contracts. $\square$

3. Silent Open-ended Search

We denote by $\left(\mu_1^S, \mu_2^S\right)$ the provider's equilibrium effort under the Silent Open-ended search in the first and second rounds, respectively. In addition, we denote by U_t^S and V_t^S, the provider's and client's payoff to-go, respectively, under the Silent Open-ended search in round t for $t \in \{1, 2\}$.

Proof of Proposition 6: At the end of the first round and when v_1 is observed by the client, the client and provider simultaneously choose their equilibrium actions. In particular, the client decides whether to stop or continue the search and the provider chooses his effort level in case of continuation. In particular, they solve the following game:

$$
\begin{aligned}
v_s^S &= \max_{v \geq A} \left\{ v \mid \mathbb{E}_{\mu_2^S}\left[V_2^S\left(\mu_2^S, v\right)\right] \geq (v - F) \right\} \\
\mu_2^S &= \arg \max_{\mu \in [\mu_0, 1]} U_2\left(\mu, v_s^S\right).
\end{aligned}
$$

In particular, when $v_1 \geq A$, the client chooses to continue or stop the search by solving the following maximization problem:

$$
\begin{aligned}
V_2^S &= \max\left\{(v_1 - F), \mathbb{E}_{\mu_2^S}\left[V_2^S\left(\mu_2^S\right)\right]\right\} \\
&= \max\left\{(v_1 - F), -c_I - w + p_{\mu_2^S}(v_2 \geq v_1) \cdot \left(\mathbb{E}_{\mu_2^S}[v_2 \mid v_2 \geq v_1] - F\right) + \left(1 - p_{\mu_2^S}(v_2 \geq v_1)\right) \cdot (v_1 - F)\right\}.
\end{aligned}
$$

By the memory-less property of the exponential probability distribution, $\mathbb{E}_{\mu_2^{O+}}[v_2 \mid v_2 \geq v_1] = v_1 + k$. We therefore obtain

$$
\begin{aligned}
V_2^S &= \max\left\{(v_1 - F), -c_I - w + p_{\mu_2^S}(v_2 \geq v_1) \cdot \left(\mathbb{E}_{\mu_2^S}[v_2 \mid v_2 \geq v_1] - F\right) + \left(1 - p_{\mu_2^S}(v_2 \geq v_1)\right) \cdot (v_1 - F)\right\} \\
&= \max\left\{(v_1 - F), -c_I - w + \mu_2^S \cdot k \cdot e^{-v_1/k} + v_1 - F\right\},
\end{aligned}
$$

which implies the client stops the search if and only if $-c_I - w + \mu_2^S \cdot k \cdot e^{-v_1/k} \leq 0$, which is decreasing in v_1. Hence, there exists a unique threshold $v_s^S\left(\mu_2^S\right) \doteq k \cdot \ln\left(\frac{\mu_2^S k}{c_I + w}\right)$ such that $-c_I - w + \mu_2^S \cdot k \cdot e^{-v_1/k} \leq 0$ if and only if $v_1 \geq v_s^S$. In addition, because by assumption $v_1 \geq A$, the client stops the search if $v_1 \geq \max\left\{A, v_s^S\left(\mu_2^S\right)\right\}$ and continue the search if $A \leq v_1 < \max\left\{A, v_s^S\left(\mu_2^S\right)\right\}$.

Similarly, the provider chooses μ_2^S as follows:

$$
\begin{aligned}
\mu_2^S &= \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + p(v_2 \geq A) \cdot F + (1 - p(v_2 \geq A)) \cdot p\left(v_1 \geq A \mid v_1 < v_s^S\right) \cdot F \\
&\quad \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + p(v_2 \geq A) \cdot F + (1 - p(v_2 \geq A)) \cdot \left(\frac{p\left(A \leq v_1 < v_s^S\right)}{p(v_1 < v_s^S)}\right) \cdot F \\
&= \arg\max_{\mu \in [\mu_0, 1]} -c\mu^2 + \mu e^{-A/k} \cdot F + \left(1 - \mu e^{-A/k}\right) \cdot \left(\frac{\mu_1^S e^{-A/k} - \mu_1^S e^{-v_s^S/k}}{1 - \mu_1^S e^{-V_s^S/lk}}\right) \cdot F,
\end{aligned}
$$

which is concave in μ. Thus, by the first order conditions, we obtain that μ_2^S solves

$$
-2c\mu + e^{-A/k} \cdot F - e^{-A/k} \cdot F \cdot \frac{\mu_1^S \left(e^{-A/k} - e^{-v_s^S(\mu)/k}\right)}{1 - \mu_1^S \cdot e^{-v_s^S(\mu)/k}} = 0.
$$

Replacing $v_s^S\left(\mu_2^S\right) \doteq k \cdot \ln\left(\frac{\mu_2^S k}{c_I + w}\right)$, we obtain that μ_2^S solves $\chi\left(\mu\right) = 0$, where

$$
\chi\left(\mu\right) \doteq -2c\mu + e^{-A/k} \cdot F \cdot \left(1 - \frac{\mu_2^S \left(k\mu e^{-A/k} - (c_I + w)\right)}{k\mu - \mu_1^S \left(c_I + w\right)}\right).
$$

The function $\chi\left(\mu\right)$ is quadratic in μ and has only two solutions of 0 and $\frac{e^{-A/k} F\left(1 - \mu_1^S e^{-A/k}\right)}{2c} + \frac{\mu_1(c_I + w)}{k}$. Because by assumption $\mu_2^S \geq \mu_0 > 0$, we obtain $\mu_2^S(\mu_1^S) = \max\left\{\mu_0, \frac{e^{-A/k} F\left(1 - \mu_1^S e^{-A/k}\right)}{2c} + \frac{\mu_1^S(c_I + w)}{k}\right\}$, which is a function of μ_1^S. Accordingly, the provider's payoff in the second round is $U_2^S(\mu_1^S) = -c\left(\mu_2^S\right)^2 + w + \mu_2^S e^{-A/k} \cdot F + \left(1 - \mu_2^S e^{-A/k}\right) \cdot \left(\frac{\mu_1^S \cdot e^{-A/k} - \mu_1^S \cdot e^{-v_s^S(\mu)/k}}{1 - \mu_1^S \cdot e^{-v_s^S(\mu)/k}}\right) \cdot F.\square$

Lemma 10. *The provider's effort in the second round of the Silent search is smaller than his effort in the second round of the Committed search (i.e., $\mu_2^S \leq \mu_2^C$).*

Proof. Using Prop. **1** and comparing μ_2^S and μ_2^C, we obtain $\mu_2^S - \mu_2^C = \mu_1^S \cdot \left(\frac{(c_I + w)}{k} - \frac{\left(e^{-A/k}\right)^2 F}{2c}\right)$. Thus, given that $\mu_1^S \geq \mu_0 > 0$, we obtain $\mu_2^S - \mu_2^C \leq 0$ if and only if $\frac{(c_I + w)}{k} - \frac{\left(e^{-A/k}\right)^2 F}{2c} \leq 0$. By Proposition **6**, $v_s^S\left(\mu_2^S\right) \doteq k \cdot \ln\left(\frac{\mu_2^S k}{c_I + w}\right)$. Thus, $e^{-v_s^S/k} = \frac{c_I + w}{\mu_2^S k} \geq \frac{c_I + w}{k}$ for any $\mu_2^S \in [\mu_0, 1]$. We obtain

$$
\begin{aligned}
\mu_2^S - \mu_2^C &= \mu_1^S \cdot \left(\frac{(c_I + w)}{k} - \frac{\left(e^{-A/k}\right)^2 F}{2c}\right) \\
&\leq \mu_1^S \cdot \left(e^{-v_s^S/k} \mu_2^S - e^{-A/k} \mu_2^C\right) \\
&\leq \mu_1^S e^{-A/k} \cdot \left(\mu_2^S - \mu_2^C\right),
\end{aligned}
\tag{6.5}
$$

in which the last inequality holds because $v_s^S \geq A$, i.e., $e^{-v_s^S/k} \leq e^{-A/k}$. We next show the result by contradiction. Suppose $\mu_2^S - \mu_2^C > 0$. Then, (3.1) implies that $\mu_1^S e^{-A/k} \geq 1$, a contradiction. Thus, for any

$\mu_1^S \leq 1$, $\mu_2^S - \mu_2^C \leq 0$.   □

Lemma 11. *Suppose $\mu_2^S = \frac{e^{-A/k}F}{2c} + \mu_1^S \cdot \left(\frac{(c_I + w)}{k} - \frac{\left(e^{-A/k}\right)^2 F}{2c} \right)$. Then, μ_2^S is linear and decreasing in μ_1^S.*

Proof. Taking the derivative of μ_2^S w.r.t. μ_1^S, we obtain $\frac{d\mu_2^S}{d\mu_1^S} = \frac{(c_I + w)}{k} - \frac{\left(e^{-A/k}\right)^2 F}{2c}$. We next show that $\frac{d\mu_2^S}{d\mu_1^S} \leq 0$. By Proposition **6**, $v_s^S\left(\mu_2^S\right) \doteq k \cdot \ln\left(\frac{\mu_2^S k}{c_I + w} \right)$. Thus, $e^{-v_s^S/k} = \frac{c_I + w}{\mu_2^S k} \geq \frac{c_I + w}{k}$ for any $\mu_2^S \in [\mu_0, 1]$. We obtain

$$\frac{(c_I + w)}{k} - \frac{\left(e^{-A/k}\right)^2 F}{2c} \quad \leq \quad e^{-v_s^S/k}\mu_2^S - e^{-A/k}\mu_2^D$$
$$\leq \quad e^{-A/k}\left(\mu_2^S - \mu_2^D \right) \leq 0,$$

in which the first inequality holds because $v_s^S \geq A$, i.e., $e^{-v_s^S/k} \leq e^{-A/k}$, and the second inequality holds by Lemma 10.   □

Proof of Proposition 7: (i) When $\mu_2^S = \mu_0$, we have $v_s^S\left(\mu_2^S\right) \doteq k \cdot \ln\left(\frac{\mu_0 k}{c_I + w} \right)$ and $U_2^S(\mu_1^S) = -c\left(\mu_0\right)^2 + w + \mu_0 e^{-A/k} \cdot F + \left(1 - \mu_0 e^{-A/k}\right) \cdot \left(\frac{\mu_1^S \cdot e^{-A/k} - \mu_1^S \cdot e^{-v_s^S(\mu)/k}}{1 - \mu_1^S \cdot e^{-v_s^S(\mu)/k}} \right) \cdot F$. Then, the provider chooses μ_1^S by solving the following:

$$\mu_1^S = \arg \max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p(v_1 \geq v_s^S) \cdot F + \left(1 - p(v_1 \geq v_s^S)\right) \cdot U_2^S(\mu)$$
$$= \arg \max_{\mu \in [\mu_0, 1]} -c\mu^2 + \left(e^{-A/k}F\left(1 - \mu_0 e^{-A/k}\right) + \left(\frac{c_I + w}{\mu_0 k} \right)\left(c\mu_0^2 - w\right) \right)\mu + 2w + \mu_0 e^{-A/k}F - c\mu_0^2,$$

which is concave in μ. Thus, by the first order conditions and the fact that $\mu_1^S \geq \mu_0$, we obtain $\mu_1^S = \max\left\{ \mu_0, \frac{e^{-A/k}F\left(1 - \mu_0 e^{-A/k}\right) + \left(\frac{c_I + w}{\mu_0 k} \right)\left(c\mu_0^2 - w\right)}{2c} \right\}$.

(ii) When $\mu_2^S = \frac{e^{-A/k}F}{2c} + \mu_1^S \cdot \left(\frac{(c_I + w)}{k} - \frac{\left(e^{-A/k}\right)^2 F}{2c} \right)$, we have $v_s^S(\mu) \doteq k \cdot \ln\left(\frac{\mu_2^S(\mu) k}{c_I + w} \right)$ and $U_2^S(\mu) = -c \cdot \left(\mu_2^S(\mu)\right)^2 + \mu_2^S(\mu) \cdot e^{-A/k} \cdot F + \left(1 - \mu_2^S(\mu) \cdot e^{-A/k}\right) \cdot \left(\frac{\mu e^{-A/k} - \mu e^{-v_s^S(\mu)/k}}{1 - \mu e^{-v_s^S(\mu)/lk}} \right) \cdot F$. Then, the provider chooses μ_1^S by solving the following:

$$\mu_1^S = \arg \max_{\mu \in [\mu_0, 1]} U_1^S(\mu)$$
$$= \arg \max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + p(v_1 \geq v_s^S(\mu)) \cdot F + \left(1 - p(v_1 \geq v_s^S(\mu))\right) \cdot U_2^S(\mu)$$
$$= \arg \max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + \mu e^{-v_s^S(\mu)}F + \left(1 - e^{-v_s^S(\mu)}\right) \cdot U_2^S(\mu)$$
$$= \arg \max_{\mu \in [\mu_0, 1]} -c\mu^2 + w + \mu\left(\frac{c_I + w}{\mu_2^S(\mu) k} \right)F + \left(1 - \mu\left(\frac{c_I + w}{\mu_2^S(\mu) k} \right)\right)$$
$$\cdot \left(-c \cdot \left(\mu_2^S(\mu)\right)^2 + \mu_2^S(\mu) \cdot e^{-A/k} \cdot F + \left(1 - \mu_2^S(\mu) \cdot e^{-A/k}\right) \cdot \left(\frac{\mu e^{-A/k} - \mu e^{-v_s^S(\mu)/k}}{1 - \mu e^{-v_s^S(\mu)/lk}} \right) \cdot F \right) (6.6)$$

which is obtained by replacing $v_s^S(\mu)$ and $U_2^S(\mu)$ and simplifying the expressions. We first show that U_1^S, as in (6.6), is concave in $\mu \in [0, 1]$ and has a unique maximum point.

By Lemma 11, we obtain $\frac{d\mu_2^S(\mu)}{d\mu} = Z = \frac{(c_I+w)}{k} - \frac{\left(e^{-A/k}\right)^2 F}{2c} \leq 0$. Moreover, by assumption $\frac{\left(e^{-A/k}\right)^2 F}{2c} \leq 1$ (which guarantees efforts are interior), and $\frac{(c_I+w)}{k} = \mu_2^S e^{-v_s^S} \leq 1$. Thus, we obtain $\frac{d\mu_2^S(\mu)}{d\mu} \geq -1$, i.e., $Z \in [-1.0]$ and is independent of μ. Taking the second derivative of $U_1^S(\mu)$ in (6.6) w.r.t. μ, we obtain

$$
\begin{aligned}
\frac{d^2 U_1^{SD}(\mu)}{d\mu^2} \;=\;& \frac{-2}{\left(\mu_2^S(\mu)\right)^3 k} \cdot \left[Z^2 \cdot \left(ck \left(\mu_2^S(\mu)\right)^3 + \mu w \left(c_I + w\right) \right) + 2ck \left(\mu_2^S(\mu)\right)^3 \right. \\
&\left. + Z \cdot \mu_2^S(\mu) \left(F \cdot \left(\mu_2^S(\mu)\right)^2 \left(e^{-A/k}\right)^2 k - (w + c_I) \left(c \cdot \left(\mu_2^S(\mu)\right)^2 + w \right) \right) \right].
\end{aligned}
$$

Replacing $\left(e^{-A/k}\right)^2 F = 2c \cdot \left(\frac{(c_I+w)}{k} - Z \right)$, we obtain

$$
\begin{aligned}
\frac{d^2 U_1^{SD}(\mu)}{d\mu^2} \;=\;& \frac{-2}{\left(\mu_2^S(\mu)\right)^2} \cdot \left[Z^2 \cdot \left(-c \cdot \left(\mu_2^S(\mu)\right)^2 + \mu w \left(\frac{c_I+w}{k\mu_2^S(\mu)} \right) \right) + c \left(\mu_2^S(\mu)\right)^2 \right. \\
&\left. + Z \left(\mu_2^S(\mu) \left(\frac{w+c_I}{\mu_2^S(\mu)k} \right) \left(c \cdot \left(\mu_2^S(\mu)\right)^2 - w \right) \right) \right].
\end{aligned}
$$

We consider two cases:

(a) Suppose $c \cdot \left(\mu_2^S(\mu)\right)^2 - w \leq 0$. Then, (i) if $-c \left(\mu_2^S(\mu)\right)^2 + \mu w \left(\frac{c_I+w}{k\mu_2^S(\mu)} \right) \geq 0$, we obtain $\frac{d^2 U_1^{SD}(\mu)}{d\mu^2} \leq 0$, which implies $U_1^{SD}(\mu)$ is concave in $\mu \in [0, 1]$. (ii) If $-c \left(\mu_2^S(\mu)\right)^2 + \mu w \left(\frac{c_I+w}{k\mu_2^S(\mu)} \right) \leq 0$, then $\frac{d^2 U_1^{SD}(\mu)}{d\mu^2}$ is convex in Z. In addition, because $Z \in [-1, 0]$, we obtain that when $z = 0$, $\frac{d^2 U_1^{SD}(\mu)}{d\mu^2} = -2c \leq 0$ and when $z = -1$, $\frac{d^2 U_1^{SD}(\mu)}{d\mu^2} = \frac{2(w+c_I)}{\left(\mu_2^S(\mu)\right)^3 k} \left(\mu_2^S(\mu) \left(c \cdot \left(\mu_2^S(\mu)\right)^2 \right) - \left(\mu_2^S(\mu) + \mu\right) \cdot w \right) \leq 0$, because $c \cdot \left(\mu_2^S(\mu)\right)^2 - w \leq 0$ and $\mu_2^S(\mu) \leq \mu_2^S(\mu) + \mu$. Thus, $\frac{d^2 U_1^{SD}(\mu)}{d\mu^2} \leq 0$ for all $Z \in [-1, 0]$, which implies $U_1^{SD}(\mu)$ is concave in $\mu \in [0, 1]$.

(b) Suppose $c \cdot \left(\mu_2^S(\mu)\right)^2 - w > 0$. Replacing $\frac{w+c_I}{\mu_2^S(\mu)k} =$, we obtain

$$
\begin{aligned}
\frac{d^2 U_1^{SD}(\mu)}{d\mu^2} \;=\;& \frac{-2}{\left(\mu_2^S(\mu)\right)^2} \cdot \left[-Z^2 \cdot \left(c \left(\mu_2^S(\mu)\right)^2 - \mu w \left(e^{-v_s^S/k}\right) \right) + c \left(\mu_2^S(\mu)\right)^2 \right. \\
&\left. + Z \cdot \left(\mu_2^S(\mu) \left(e^{-v_s^S/k}\right) \left(\left(\mu_2^S(\mu)\right)^2 c - w \right) \right) \right] \\
\;=\;& \frac{-2}{\left(\mu_2^S(\mu)\right)^2} \cdot \left[-Z^2 \cdot \left(c \left(\mu_2^S(\mu)\right)^2 - w \right) - Z^2 \cdot \left(w - \mu w \left(e^{-v_s^S/k}\right) \right) + c \left(\mu_2^S(\mu)\right)^2 \right. \\
&\left. + Z \cdot \left(\mu_2^S(\mu) \left(e^{-v_s^S/k}\right) \left(\left(\mu_2^S(\mu)\right)^2 c - w \right) \right) \right] \\
\;=\;& \frac{-2}{\left(\mu_2^S(\mu)\right)^2} \cdot \left[Z \cdot \left(c \left(\mu_2^S(\mu)\right)^2 - w \right) \cdot \left(\mu_2^S(\mu) \left(e^{-v_s^S/k}\right) - Z \right) \right. \\
&\left. + c \left(\mu_2^S(\mu)\right)^2 - w \cdot Z^2 \cdot \left(1 - \mu \left(e^{-v_s^S/k}\right) \right) \right].
\end{aligned}
$$

Because $Z \in [-1, 0]$ (i.e., $Z^2 \in [0, 1]$) and $\left(1 - \mu \left(e^{-v_s^S/k}\right) \right) \in [0, 1]$, we obtain $c \left(\mu_2^S(\mu)\right)^2 - w \cdot Z^2 \cdot$

$\left(1 - \mu\left(e^{-v_s^S/k}\right)\right) \geq c\left(\mu_2^S(\mu)\right)^2 - w$. Thus,

$$
\begin{aligned}
\frac{d^2 U_1^{SD}(\mu)}{d\mu^2} &\leq \frac{-2}{\left(\mu_2^S(\mu)\right)^2} \cdot \left[\left(c\left(\mu_2^S(\mu)\right)^2 - w\right) \cdot \left(Z \cdot \mu_2^S(\mu)\left(e^{-v_s^S/k}\right) - Z^2 + 1\right)\right] \\
&= \frac{-2}{\left(\mu_2^S(\mu)\right)^2} \cdot \left[\left(c\left(\mu_2^S(\mu)\right)^2 - w\right) \cdot \left(Z \cdot \left(\mu_2^S(\mu)\left(e^{-v_s^S/k}\right) - Z\right) + 1\right)\right] \\
&= \frac{-2}{\left(\mu_2^S(\mu)\right)^2} \cdot \left[\left(c\left(\mu_2^S(\mu)\right)^2 - w\right) \cdot \left(Z \cdot \left(\frac{\left(e^{-A/k}\right)^2 F}{2c}\right) + 1\right)\right].
\end{aligned}
$$

By assumption, $\frac{\left(e^{-A/k}\right)^2 F}{2c} \leq 1$ to guarantee that efforts are interior. In addition, $Z \in [-1, 0]$. Thus, $Z \cdot \left(\frac{\left(e^{-A/k}\right)^2 F}{2c}\right) + 1 \geq 0$. Thus, we obtain $\frac{d^2 U_1^{SD}(\mu)}{d\mu^2} \leq 0$, which implies that $U_1^{SD}(\mu)$ is concave in $\mu \in [0, 1]$.

Knowing that $U_1^{SD}(\mu)$ is concave in $\mu \in [0, 1]$, by the first order condition and the fact that $\mu_1^S \geq \mu_0$, we obtain that $\mu_1^S = \max\{\mu_0, \hat{\mu}\}$, in which $\hat{\mu}$ is the unique solution to

$$
\begin{aligned}
\frac{dU_1^{SD}(\mu)}{d\mu} &= 2c\mu Z^2 + \left(Fe^{-A/k} - \mu \cdot \left(\mu_2^S(\mu)\right)^{-1} \cdot e^{-v_s^S(\mu)}\left(\left(\mu_2^S(\mu)\right)^2 - w\right)\right) \cdot Z \\
&\quad + 2Fc\mu e^{-A/k} e^{-v_s^S(\mu)/k}\left(\left(\mu_2^S(\mu)\right)^2 + w\right) = 0. \square
\end{aligned}
$$

Proof of Proposition 8: By Propositions 3 and 6, $v_s^S = k \cdot \ln\left(\frac{\mu_0 \cdot k}{c_I + w}\right)$ and $v_s^O = k \cdot \ln\left(\frac{\mu_2^S \cdot k}{c_I + w}\right)$, because by assumption $\mu_2^S \geq \mu_0$, we obtain $v_s^O \geq v_s^S$. In addition, because v_s^O is monotone increasing in k, there exists a unique threshold $\bar{k}$ such that $v_s^O \geq A$ if and only if $k \geq \bar{k}$, where $\bar{k}$ is the unique solution to $\bar{k} \cdot \ln\left(\frac{\mu_0 \cdot \bar{k}}{c_I + w}\right) = 0. \square$

Proof of Proposition 9: (i) By Propositions 1 and 4, we have $\mu_2^{O-} = \mu_2^C$, and by Lemma 10, $\mu_2^C \geq \mu_2^S$. In addition, by Proposition 4, $\mu_2^{O+} = \mu_0 \leq \mu_2^S$.

(ii) By Proposition 1, $\mu_1^C = \max\left\{\mu_0, \frac{\left(F - U_2^C\right) \cdot e^{-A/k}}{2c}\right\}$, and by Proposition 4,

$$
\mu_1^O = \max\left\{\mu_0, \frac{\left(e^{-A/k} - e^{-\max\{A, v_s^O\}/k}\right) \cdot \left(-c\mu_0^2 + w\right) + e^{-A/k} \cdot \left(F - U_2^{O-}\right)}{2c}\right\}.
$$ In addition, because $\mu_2^C = \mu_2^{O-}$, we obtain $U_2^C = U_2^{O-}$. Finally, because $\max\{A, v_s^O\} \geq A$, we obtain $\left(e^{-A/k} - e^{-\max\{A, v_s^O\}/k}\right) \geq 0$. We therefore obtain, $\mu_1^O \geq \mu_1^C$ if and only if $-c\mu_0^2 + w \geq 0. \square$